HIGHER

4.19H Probability: Higher Tier Practice

EDEXCEL 1MA1 · Calculator allowed · about 75 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A single fair six-sided die is rolled once. Work out the probability that the result is an even number greater than 2.

(Total for Question 1 is 2 marks)

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2 A bag contains 3 red counters, 5 blue counters and 2 green counters. Two counters are drawn at random without replacement. Work out the probability that both counters are blue. Give your answer as a fraction in simplest form.

(Total for Question 2 is 3 marks)

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3 A spinner has three equal sectors labelled A, B and C. The spinner is spun twice. Complete the tree diagram probabilities and then find the probability that the outcome is A on the first spin and not C on the second spin.

(Total for Question 3 is 4 marks)

4 A fair coin is tossed 3 times. Work out the probability of getting exactly two heads. Show your method.

(Total for Question 4 is 5 marks)

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5 A bag contains counters numbered 1 to 5. A counter is chosen at random. Find the probability that the number is a prime number or an even number. State clearly any numbers counted twice and show how you avoid double counting.

(Total for Question 5 is 6 marks)

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6 A box contains 4 white balls and 6 black balls. Two balls are selected at random with replacement. Let X be the number of white balls selected. (a) State the probability distribution for X. (b) Find E(X), the expected number of white balls in the two draws.

(a) State the probability distribution for X (X can be 0, 1 or 2). **(3)**

(b) Find E(X). **(3)**

(Total for Question 6 is 6 marks)

7 A biased coin lands heads with probability 0.6. The coin is tossed 5 times. (a) Find the probability of getting exactly 3 heads. (b) State the most likely number of heads and justify your answer briefly.

(a) Find the probability of getting exactly 3 heads. **(5)**

(b) State the most likely number of heads and justify briefly. **(3)**

(Total for Question 7 is 8 marks)

8 A game gives a prize according to the following rule. A bag contains tokens labelled with values £1, £2 and £5. The probabilities of drawing each are proportional to 1, 2 and 1 respectively. A player draws one token at random and receives that amount. The player may then choose to redraw once: if they redraw they must accept the second token. (a) Find the probability of drawing each value on a single draw. (b) Determine the expected winning if the player adopts the strategy: keep the first draw only if it is at least £2; otherwise redraw. Show all working and give the expected value to the nearest pence. (c) Is this strategy better than always keeping the first draw? Justify with a comparison.

(a) Find the probability of drawing each value on a single draw. **(2)**

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(b) Determine the expected winning with the strategy: keep first draw only if it is at least £2; otherwise redraw. Show working and give expected value to nearest pence. **(5)**

(c) Is this strategy better than always keeping the first draw? Justify with a comparison. **(3)**

(Total for Question 8 is 10 marks)

Mark scheme · 4.19H Probability: Higher Tier Practice

Question 1

- **M1** identify favourable outcomes 4 and 6 or method showing count 2/6
- **A1** 1/3 cao
- **Answer: 1/3**

Question 2

- **M1** use conditional probability: 5/10 x 4/9 or equivalent method
- **A1** 20/90 = 2/9 cao
- **B1** fully reduced fraction 2/9
- **Answer: 2/9**

Question 3

- **M1** correct probabilities for branches: 1/3 for each first spin and 1/3 each for second spin
- **M1** identify probability for A then not C as 1/3 x (1 - 1/3) or 1/3 x 2/3
- **A1** 2/9 cao
- **B1** explicit statement 1/3 x 2/3 = 2/9
- **Answer: 2/9**

Question 4

- **M1** use combinations or list outcomes: number of ways to get 2 heads = 3 (HHT, HTH, THH) or calculate 3 choose 2
- **M1** probability method: 3 x $(1/2)^3$ or (3 choose 2) x $(1/2)^3$
- **A1** 3/8 cao
- **B1** explicit enumeration or binomial reasoning given
- **B1** final answer stated as 3/8
- **Answer: 3/8**

Question 5

- **M1** identify primes in 1-5: 2,3,5 and evens: 2,4
- **M1** state overlap: number 2 counted in both sets
- **M1** use formula P(A or B) = P(A) + P(B) - P(A and B) or equivalent
- **A1** compute: (3/5) + (2/5) - (1/5) = 4/5 cao
- **B1** explain avoidance of double counting or show Venn diagram style reasoning
- **B1** final answer 4/5 stated
- **Answer: 4/5**

Question 6

- (a) **M1** identify p(white) = 4/10 = 2/5 and p(black) = 3/5
- (a) **M1** p(0) = $(3/5)^2$ = 9/25 and p(2) = $(2/5)^2$ = 4/25
- (a) **A1** p(1) = 1 - p(0) - p(2) = 12/25 cao
- **(a) Answer: P(X=0)=9/25, P(X=1)=12/25, P(X=2)=4/25**
- (b) **M1** use E(X)=sum xP(x) or note linearity E(X)=2 x p(white)
- (b) **A1** compute E(X)=0*(9/25)+1*(12/25)+2*(4/25)=20/25
- (b) **A1** final simplified value 4/5 cao
- **(b) Answer: 4/5**

Question 7

- (a) **M1** use binomial formula: $C(5,3)*(0.6)^3*(0.4)^2$
- (a) **M1** compute combination 10 or show numeric substitution
- (a) **M1** numerical evaluation to 3 significant figures or exact decimal
- (a) **A1** 0.3456 cao
- (a) **B1** final answer stated with correct rounding if used
- **(a) Answer: 0.3456**
- (b) **M1** identify mean of binomial is np = 5*0.6 = 3
- (b) **A1** most likely number is 3 (mode for binomial when (n+1)p is integer state either 3 or 3) cao
- (b) **B1** brief justification: compare probabilities or cite that mode ~ floor((n+1)p) = 3
- **(b) Answer: 3, because the mean np = 3 and the binomial mode is 3 (highest probability).**

Question 8

- (a) **M1** recognise probabilities proportional to 1:2:1 so total parts =4
- (a) **A1** P(1)=1/4, P(2)=2/4=1/2, P(5)=1/4 cao
- **(a) Answer: P(£1)=1/4, P(£2)=1/2, P(£5)=1/4**
- (b) **M1** identify cases: keep if first draw £2 or £5; redraw if £1
- (b) **M1** compute expected value on redraw = E(second draw) = 1*(1/4)+2*(1/2)+5*(1/4) = (1/4)+(1)+(5/4)= (1/4+1+1.25)=2.5
- (b) **M1** compute total expectation: P(first £1)*E(second) + P(first £2)*2 + P(first £5)*5
- (b) **A1** substitute: (1/4)*2.5 + (1/2)*2 + (1/4)*5 = 0.625 + 1.0 + 1.25 = 2.875
- (b) **A1** final answer £2.88 to nearest pence cao
- **(b) Answer: £2.88**
- (c) **M1** compute expected value if always keep first draw = E(first) = 2.5
- (c) **A1** compare 2.875 > 2.5 and state strategy gives higher expectation
- (c) **B1** conclude strategy is better, justify by difference £0.375 or 37.5p
- **(c) Answer: Yes. Always keeping first draw has expected value £2.50. The strategy gives £2.875, so it is better by £0.375 (37.5p).**