HIGHER

4.3H Prime Factors, HCF and LCM: Higher Tier Practice

Total Marks

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 75 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Express 2310 as a product of prime factors.

(Total for Question 1 is 2 marks)

2 Work out the highest common factor (HCF) of 378 and 630.

(Total for Question 2 is 2 marks)

3 Find the least common multiple (LCM) of 40 and 96 using prime factors.

(Total for Question 3 is 3 marks)

4 The number A = 2^4 x 3^2 x 5 and the number B = 2^2 x 3^3 x 7.
(a) Find HCF(A, B).
(b) Find LCM(A, B).

(a) Find HCF(A, B). **(2)**

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(b) Find LCM(A, B). **(2)**

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(Total for Question 4 is 4 marks)

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5 Show that for any two positive integers a and b with prime factorisations a = product p_i^{α_i} and b = product p_i^{β_i} (over the same primes p_i), the product a x b equals HCF(a, b) x LCM(a, b). You must give a reason referring to prime exponents. This is a 'show that' question.

(Total for Question 5 is 4 marks)

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6 Two buses leave the same stop at 08:00. One bus completes a lap every 18 minutes, the other every 30 minutes. At what time after 08:00 will both buses next arrive at the stop together? Give your answer in hours and minutes.

(Total for Question 6 is 3 marks)

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7 Find the smallest positive integer n such that 360 x n is a perfect cube.

(Total for Question 7 is 3 marks)

8 Work out the largest three-digit number that is divisible by both 18 and 28.

(Total for Question 8 is 2 marks)

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9 Let $x = 2^4 \times 3^1 \times 5^2$.

(a) Find the number of positive divisors of x.

(b) How many of these divisors are multiples of 5?

(a) Find the number of positive divisors of x. **(2)**

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(b) How many of these divisors are multiples of 5? **(2)**

.....

(Total for Question 9 is 4 marks)

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10 Find all positive integers n such that HCF(n, 18) = 6 and LCM(n, 18) = 90. Show your working.

(a) Find all positive integers n satisfying the conditions. **(2)**

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(Total for Question 10 is 2 marks)

Mark scheme · 4.3H Prime Factors, HCF and LCM: Higher Tier Practice

Question 1

- **M1** correct division tree or primes shown, e.g. 2310 = 2 x 3 x 5 x 7 x 11 (method)
- **A1** 2310 = 2 x 3 x 5 x 7 x 11 cao
- **Answer: 2 x 3 x 5 x 7 x 11 (2310 = 2 x 3 x 5 x 7 x 11)**

Question 2

- **M1** correct prime factorisation or identification of common prime powers, e.g. 378 = 2 x 3^3 x 7, 630 = 2 x 3^2 x 5 x 7
- **A1** HCF = 2 x 3^2 x 7 = 126 cao
- **Answer: 126**

Question 3

- **M1** correct prime factorisations, e.g. 40 = 2^3 x 5, 96 = 2^5 x 3
- **M1** take highest powers: 2^5 x 3 x 5
- **A1** LCM = 32 x 3 x 5 = 480 cao
- **Answer: 480**

Question 4

- (a) **M1** choose minimum powers: 2^2 x 3^2 oe
- (a) **A1** HCF = 4 x 9 = 36 cao
- **(a) Answer: 36**
- (b) **M1** take highest powers: 2^4 x 3^3 x 5 x 7 oe
- (b) **A1** LCM = 16 x 27 x 5 x 7 = 15120 cao
- **(b) Answer: 15120**

Question 5

- **M1** express HCF exponents as $\min(\alpha_i, \beta_i)$ and LCM exponents as $\max(\alpha_i, \beta_i)$
- **M1** note that for each i, $\alpha_i + \beta_i = \min(\alpha_i,\beta_i) + \max(\alpha_i,\beta_i)$
- **M1** multiply over all primes to get a x b = product $p_i^{\alpha_i+\beta_i}$ = product $p_i^{\min+\max}$ = HCF x LCM (method)
- **A1** clear conclusion: therefore a x b = HCF(a,b) x LCM(a,b) cso
- **Answer: For each prime p_i, exponent in HCF = $\min(\alpha_i,\beta_i)$, exponent in LCM = $\max(\alpha_i,\beta_i)$. $\alpha_i + \beta_i = \min + \max$, so multiplying $p_i^{\alpha_i} p_i^{\beta_i} = p_i^{\min} p_i^{\max}$. Product over all primes gives a x b = HCF x LCM. Hence shown.**

Question 6

- **M1** find LCM of 18 and 30, e.g. 18 = 2 x 3^2, 30 = 2 x 3 x 5 so LCM = 2 x 3^2 x 5 = 90
- **M1** convert 90 minutes to hours and minutes
- **A1** 90 minutes = 1 hour 30 minutes, so both buses meet at 09:30 cao
- **Answer: 09:30 (1 hour 30 minutes after 08:00)**

Question 7

- **M1** prime factorise 360 = 2^3 x 3^2 x 5^1
- **M1** choose multipliers to make exponents multiples of 3: need 3^1 and 5^2
- **A1** n = 3 x 25 = 75 cao
- **Answer: 75**

Question 8

- **M1** find LCM(18,28) = 2^2 x 3^2 x 7 = 252
- **A1** largest three-digit multiple is 252 x 3 = 756 cao
- **Answer: 756**

Question 9

- (a) **M1** use formula (exponents + 1) product: (4+1)(1+1)(2+1) method
- (a) **A1** number of divisors = 5 x 2 x 3 = 30 cao
- **(a) Answer: 30**
- (b) **M1** count divisors that include at least one factor 5: treat exponent of 5 as 1 or 2 -> (4+1)(1+1) x (choices for 5 exponent 2) or more direct: (4+1)(1+1) where exponent of 5≥1 gives 5 x 2 x 2? (method shown)
- (b) **A1** divisors multiple of 5 = 5 x 2 x 2 = 20 cao
- **(b) Answer: 20**

Question 10

- (a) **M1** use relation n x 18 = HCF x LCM = 6 x 90 = 540 so n = 540 / 18
- (a) **A1** n = 30 cao
- **(a) Answer: 30**