



4.5H Inequalities: Higher Tier Practice

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 75 minutes

Total Marks

Name: _____ Date: ____ / ____ / ____

Answer ALL questions. Show all your working.

- 1** Write down the inequality shown on this number line.
A filled circle at -2 and an arrow to the right from -2 to infinity.

(Total for Question 1 is 1 mark)

- 2** Solve the inequality $3x - 5 < 10$. Show all steps.

(Total for Question 2 is 2 marks)

- 3** Solve the pair of inequalities and give the solution as a single inequality:
 $2x + 1 \geq 5$ and $x - 4 < 3$.

(Total for Question 3 is 2 marks)

- 4** Solve the inequality $4 - 2x \leq 10$. Give your answer in the form $x \geq$ or $x \leq$.

(Total for Question 4 is 3 marks)

- 5 Solve the quadratic inequality $x^2 - 5x + 6 < 0$. Show how you find the critical values and give the solution in interval notation.

(Total for Question 5 is 4 marks)

- 6 Solve the inequality $(x+1)/(x-2) \geq 0$. State clearly any values excluded from the solution and give the final solution as a single inequality or union of intervals.

(Total for Question 6 is 5 marks)

- 7 Priya has two machines. Machine A produces between 20 and 35 items per hour inclusive. Machine B produces at least 15 items per hour but fewer than 28 items per hour. Let a be the rate of A and b be the rate of B. Write inequalities for a and b , then find the range of possible values for $a + b$. Give your answer as an inequality.

(Total for Question 7 is 6 marks)

- 8** Show that for all real x , if $2x - 3 < 7$ then $x < 5$. Then determine whether the converse statement is true: "If $x < 5$ then $2x - 3 < 7$ ". Give a brief justification for your answer.

(Total for Question 8 is 9 marks)

- 9** The inequality $2x^2 - 7x - 15 < 0$ is given.

(a) Show that $2x^2 - 7x - 15 = (2x + 3)(x - 5)$ and hence solve $2x^2 - 7x - 15 < 0$, giving your answer in inequality form.

(b) Hence, or otherwise, find how many integer values of x satisfy $2x^2 - 7x - 15 < 0$. Explain your reasoning clearly.

(a) Show that $2x^2 - 7x - 15 = (2x + 3)(x - 5)$ and hence solve $2x^2 - 7x - 15 < 0$, giving your answer in inequality form. (3)

(b) Hence, or otherwise, find how many integer values of x satisfy $2x^2 - 7x - 15 < 0$. Explain your reasoning clearly. (5)

(Total for Question 9 is 8 marks)

Mark scheme · 4.5H Inequalities: Higher Tier Practice

Question 1

- **B1** $x \geq -2$ cao
- **Answer:** $x \geq -2$

Question 2

- **M1** add 5 to both sides: $3x < 15$ (method)
- **A1** $x < 5$ cao
- **Answer:** $x < 5$

Question 3

- **M1** solve each: $2x \geq 4 \rightarrow x \geq 2$; $x < 7$ (method for each inequality)
- **A1** $2 \leq x < 7$ cao
- **Answer:** $2 \leq x < 7$

Question 4

- **M1** subtract 4 from both sides: $-2x \leq 6$ (method)
- **M1** divide both sides by -2 and reverse the inequality sign (method)
- **A1** $x \geq -3$ cao
- **Answer:** $x \geq -3$

Question 5

- **M1** factorise: $x^2 - 5x + 6 = (x - 2)(x - 3)$ (method)
- **M1** identify critical values $x = 2$ and $x = 3$ (method)
- **M1** use sign analysis on intervals $(-\infty, 2)$, $(2, 3)$, $(3, \infty)$ to determine where expression is negative
- **A1** $2 < x < 3$ cao
- **Answer:** $2 < x < 3$

Question 6

- **M1** identify critical points $x = -1$ (numerator zero) and $x = 2$ (denominator zero) and state $x = 2$ is excluded
- **M1** write the sign intervals: $(-\infty, -1)$, $\{-1\}$, $(-1, 2)$, $(2, \infty)$ or equivalent interval partition
- **M1** test signs or use multiplicative signs to determine where $(x+1)(x-2) \geq 0$
- **M1** decide whether $x = -1$ is included since numerator zero gives value 0, and $x = 2$ is excluded
- **A1** final solution: $(-\infty, -1] \cup (2, \infty)$ cao, with $x = 2$ excluded
- **Answer:** $x \leq -1$ or $x > 2$ (i.e. $(-\infty, -1] \cup (2, \infty)$), with $x = 2$ excluded

Question 7

- **M1** write inequalities: $20 \leq a \leq 35$ and $15 \leq b < 28$ (method)
- **M1** find minimum $a + b$ by adding minima: $20 + 15 = 35$ (method)
- **M1** find maximum $a + b$ by adding maxima as a bound: $35 + 28 = 63$ (method)
- **M1** note the strictness from $b < 28$ makes the upper bound strict so $a + b < 63$ (method)
- **A1** state final inequality $35 \leq a + b < 63$ cao
- **B1** reason that endpoints 35 and 63 follow from inclusive and exclusive bounds respectively oe
- **Answer:** $35 \leq a + b < 63$

Question 8

- **M1** add 3 to both sides of $2x - 3 < 7$ to get $2x < 10$ (method)
- **M1** divide both sides by 2 (positive) to give $x < 5$ (method)
- **A1** conclude $x < 5$ for the first part cao
- **M1** state that the converse must be tested: assume $x < 5$ (method)
- **M1** multiply the assumed inequality by 2 to get $2x < 10$ (method)
- **M1** subtract 3 to get $2x - 3 < 7$ (method)
- **A1** conclude the converse is true: if $x < 5$ then $2x - 3 < 7$ cao
- **B1** state the reason concisely: multiplication by a positive number and subtraction preserve strict inequality oe
- **B1** explicitly note domain is all real x so no hidden exceptions oe
- **Answer: First part: $x < 5$. Converse: true. Reason: assuming $x < 5$, multiply by 2 to obtain $2x < 10$ then subtract 3 to get $2x - 3 < 7$; operations preserve the strict inequality because $2 > 0$.**

Question 9

- (a) **M1** correct factorisation shown: $(2x + 3)(x - 5)$ (method)
- (a) **M1** identify roots $x = -3/2$ and $x = 5$ from factors (method)
- (a) **A1** solution: $-3/2 < x < 5$ cao
- **(a) Answer: $-3/2 < x < 5$**
- (b) **M1** use result from part (a): $-3/2 < x < 5$ (follow through)
- (b) **M1** identify integers strictly greater than -1.5 and strictly less than 5: -1, 0, 1, 2, 3, 4 (method)
- (b) **M1** count the listed integers correctly (method)
- (b) **A1** state final answer 6 integers cao
- (b) **B1** justify endpoints excluded because inequality is strict so $x = -3/2$ and $x = 5$ are not allowed oe
- **(b) Answer: There are 6 integer solutions: $x = -1, 0, 1, 2, 3, 4$.**