4.7 Sequences (Nth Term): 1-hour Pack

EDEXCEL 1MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Here are the first four terms of a sequence.
14, 10, 6, 2
Find an expression, in terms of n, for the nth term of this sequence.

(Total for Question 1 is 2 marks)

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2 The 2nd term of an arithmetic sequence is 23 and the 9th term is 65.
Find an expression for the nth term of the sequence.

(Total for Question 2 is 3 marks)

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3 The 5th term of an arithmetic sequence is 12 and the 7th term is 16.
Find an expression for the nth term of the sequence.

(Total for Question 3 is 3 marks)

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4 Work out the 6th term of the arithmetic sequence with nth term 7n - 4.

(Total for Question 4 is 2 marks)

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5 The table shows the number of dots used to make a sequence of patterns.
Pattern number (n): 1, 2, 3, 4, 5
Number of dots: 2, 7, 12, 17, ?

(a) Complete the table by finding the number of dots in Pattern 5. **(1)**

.....

(b) Find an expression, in terms of n, for the number of dots in Pattern n. **(2)**

.....

(c) Find the pattern number that has 92 dots. **(2)**

.....

(Total for Question 5 is 5 marks)

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6 A sequence of patterns is built from matchsticks. Pattern 1 is a single triangle. Each new pattern joins one more triangle onto the end of a strip, sharing one edge with the triangle before it.
Pattern 1 (one triangle) uses 3 matchsticks.
Pattern 2 (two triangles) uses 5 matchsticks.
Pattern 3 (three triangles) uses 7 matchsticks.

(a) Write down the number of matchsticks needed for Pattern 4. **(1)**

.....

(b) Find an expression, in terms of n, for the number of matchsticks in Pattern n. **(2)**

.....

(c) Zainab claims that Pattern 20 uses exactly 39 matchsticks. Show that Zainab is wrong, and find the correct number of matchsticks in Pattern 20. **(2)**

(Total for Question 6 is 5 marks)

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7 The first three terms of a sequence are 3, 8, 15.

(a) Work out the 4th term. **(1)**

.....

(b) Suggest an nth term formula for this sequence. **(2)**

.....

.....

(Total for Question 7 is 3 marks)

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8 Find the nth term of the sequence: 11, 18, 25, 32, ...
Give your answer in the form an + b.

(Total for Question 8 is 5 marks)

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9 Here are the first four terms of an arithmetic sequence.
15, 11.5, 8, 4.5

(a) Find the common difference of the sequence. **(1)**

.....

(b) Find an expression, in terms of n, for the nth term of the sequence. **(2)**

.....

(c) Find the first term in the sequence that is negative. **(3)**

.....

(Total for Question 9 is 6 marks)

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10 A sequence is defined recursively by $U_1 = 6$ and $U_{n+1} = 2U_n + 1$ for $n \geq 1$.

(a) Work out U_2 and U_3. **(2)**

(b) Show that $U_n = 7*2^{n-1} - 1$ for all $n \geq 1$. You may use proof by induction or an argument that checks the base case and shows the recurrence preserves the form. **(4)**

(Total for Question 10 is 6 marks)

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11 The nth term of a sequence is 6n - 1.
Prove algebraically that the sum of any three consecutive terms of this sequence is always a multiple of 3.

(Total for Question 11 is 3 marks)

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12 The nth term of a sequence is given by $T(n) = 2n^2 - 5n + 1$.

(a) Show that solving $T(n) = 43$ leads to the equation $2n^2 - 5n - 42 = 0$. **(1)**

(b) Solve $2n^2 - 5n - 42 = 0$ to find the value of n for which $T(n) = 43$, explaining why the other solution to the equation is not valid. **(4)**

.....

(c) Write down the term of the sequence that is equal to 43. **(1)**

.....

(Total for Question 12 is 6 marks)

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Mark scheme · 4.7 Sequences (Nth Term): 1-hour Pack

Question 1

common difference -4 **M1**

-4n + 18 **A1**

Answer: -4n + 18

Question 2

(65 - 23) / (9 - 2) = 6 **M1**

23 - 6 x 2 = 11 **M1**

6n + 11 **A1**

Answer: 6n + 11

Question 3

(16 - 12) / (7 - 5) = 2 **M1**

12 - 2 x 5 = 2 **M1**

2n + 2 **A1**

Answer: 2n + 2

Question 4

substitute n = 6 into 7n - 4 or show 7 x 6 - 4 **M1**

38 cao **A1**

Answer: 38

Question 5

(a) 22 cao **B1**

(a) Answer: 22

(b) common difference of 5 used correctly, e.g. 5n + c oe **M1**

(b) 5n - 3 oe cao **A1**

(b) Answer: 5n - 3

(c) 5n - 3 = 92 oe, or 5n = 95 (ft their expression) **M1**

(c) Pattern 19 cao **A1**

(c) Answer: Pattern 19

Question 6

(a) 9 cao **B1**

(a) Answer: 9

(b) common difference of 2 used correctly, e.g. 2n + c oe **M1**

(b) 2n + 1 oe cao **A1**

(b) Answer: 2n + 1

(c) substitutes n = 20 into their expression from part (b), e.g. 2(20) + 1 **M1**

(c) 41 (not 39), so Zainab is wrong ft cao **A1**

(c) Answer: 41 matchsticks (Zainab is wrong)

Question 7

(a) 24 cao **B1**

(a) Answer: 24

(b) recognise quadratic form with second difference 2 and write $an^2 + bn + c$ with a = 1 **M1**

(b) $n^2 + 2n$ cao **A1**

(b) Answer: $n^2 + 2n$

Question 8

identify common difference 7 and write general linear form an + b **M1**

state a = 7 and substitute n = 1 into a + b = 11 to find b **M1**

compute b = 11 - 7 = 4 **M1**

7n + 4 cao **A1**

final answer stated clearly **B1**

Answer: 7n + 4

Question 9

(a) -3.5 cao **B1**

(a) Answer: -3.5

(b) common difference of -3.5 used correctly, e.g. -3.5n + c oe **M1**

(b) 18.5 - 3.5n oe cao **A1**

(b) Answer: 18.5 - 3.5n

(c) 18.5 - 3.5n < 0 oe (ft their expression) **M1**

(c) n > 5.28... (37/7) found, leading to n = 6 **M1**

(c) 6th term, -2.5 cao **A1**

(c) Answer: The 6th term, which is -2.5

Question 10

(a) substitute $U_1 = 6$ into recurrence to get U_2, then substitute U_2 to get U_3 **M1**

(a) 13 and 27 cao **A1**

(a) Answer: $U_2 = 13$, $U_3 = 27$

(b) base case: verify n=1 gives $U_1 = 7*2^0 - 1 = 7 - 1 = 6$ **M1**

(b) assume for some k that $U_k = 7*2^{k-1} - 1$ (induction hypothesis) **M1**

(b) use recurrence: $U_{k+1} = 2U_k + 1$ and substitute hypothesis to get $2(7*2^{k-1} - 1) + 1 = 7*2^k - 1$ **M1**

(b) conclude U_{k+1} has the required form so formula holds for all $n \geq 1$ **A1**

(b) Answer: $7*2^{n-1} - 1$

Question 11

correct expressions for three consecutive terms, e.g. 6n - 1, 6n + 5, 6n + 11 oe **M1**

correctly sums and simplifies to 18n + 15 oe **M1**

factorises as 3(6n + 5) and concludes this is always a multiple of 3, since 6n + 5 is an integer cso **A1**

Answer: Proven: the sum is 3(6n + 5), which is always a multiple of 3.

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Question 12

(a) correct rearrangement of $2n^2 - 5n + 1 = 43$ to $2n^2 - 5n - 42 = 0$ cso **M1**

(a) Answer: $2n^2 - 5n - 42 = 0$ (shown)

(b) correct attempt to factorise or use the quadratic formula **M1**

(b) n = 6 found **A1**

(b) n = -3.5 (or -7/2) found as the other solution **B1**

(b) n = -3.5 rejected with valid reason (term number must be a positive integer), so n = 6 cao **A1**

(b) Answer: n = 6 (n = -3.5 is rejected as a term number must be a positive integer)

(c) the 6th term cao ft **B1**

(c) Answer: The 6th term

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