5.10H Drawing Other Graphs: Cubic/Reciprocal: Higher Tier Practice

Total Marks

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 75 minutes

Name: _______________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 Sketch the graph of $y = x^3$ for $-2 \leq x \leq 2$. Mark clearly the points where $x = -1$, 0 and 1 and give their coordinates.

(a) Mark the points for $x = -1$, 0 and 1 and write their coordinates. **(2)**

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(Total for Question 1 is 2 marks)

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2 The curve C has equation $y = (x - 2)^3 + 1$.
State the translation that maps $y = x^3$ to the curve C. Hence find the coordinates of the point on C that corresponds to $x = 2$ on $y = x^3$.

(a) State the translation. **(1)**

(b) Find the coordinates of the point on C that corresponds to the point (2, 8) on $y = x^3$. **(2)**

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(Total for Question 2 is 3 marks)

3 Sketch the reciprocal curve $y = 2/x$ for $-6 \leq x \leq 6$, excluding $x = 0$. On the axes, indicate clearly the vertical and horizontal asymptotes and state their equations.

(a) Indicate the vertical and horizontal asymptotes and give their equations. **(2)**

(b) State the coordinates of the point where the curve crosses the line $y = 1$. **(2)**

(Total for Question 3 is 4 marks)

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4 The graphs of $y = x^3 - 3x$ and $y = 2x$ are drawn on the same axes.
(a) Show that the x-coordinates of the intersection points satisfy $x^3 - 5x = 0$. (b) Hence find all intersection points.

(a) Show that the x-coordinates satisfy $x^3 - 5x = 0$. **(1)**

(b) Hence find all intersection points (give coordinates). **(3)**

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(Total for Question 4 is 4 marks)

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5 A transformed reciprocal curve has equation $y = -3/(x + 1) + 4$.
(a) State the vertical and horizontal asymptotes. (b) Find the x-intercept of this curve.

(a) State the vertical and horizontal asymptotes. **(2)**

(b) Find the x-intercept (give as an exact value). **(2)**

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(Total for Question 5 is 4 marks)

6 Show that the cubic $y = x^3 - 6x^2 + 11x - 6$ factorises as $(x - 1)(x - 2)(x - 3)$. Hence, sketch the cubic and state the x-intercepts.

(a) Show that the cubic factorises as stated. **(2)**

(b) Hence sketch the cubic and state the x-intercepts. **(2)**

(Total for Question 6 is 4 marks)

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7 Consider the family of cubics $y = x^3 + kx^2$ where k is a real constant.
(a) Show that $x = 0$ is always a root. (b) For $k = -3$ find the other two roots and classify the nature of the turning points of the cubic (local max, local min or point of inflection).

(a) Show that $x = 0$ is always a root. **(1)**

(b) For $k = -3$ find the other two roots. Then, by working out y at $x = -1, 0, 1, 2, 3$ and 4 and comparing neighbouring values, say whether the graph has a local maximum or a local minimum at $x = 0$ and at $x = 2$. **(4)**

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(Total for Question 7 is 5 marks)

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8 Show that the curve $y = 1/(x^2)$ cannot have a turning point for $x > 0$. Then explain briefly what this implies about the shape of the graph for $x > 0$.

(a) Show that the curve cannot have a turning point for $x > 0$. **(2)**

(b) Explain briefly what this implies about the shape of the graph for $x > 0$. **(2)**

(Total for Question 8 is 4 marks)

Mark scheme · 5.10H Drawing Other Graphs: Cubic/Reciprocal: Higher Tier Practice

Question 1

- (a) **B1** (-1, -1), (0, 0) and (1, 1) all correctly stated
- (a) **B1** sketch shows cubic shape (increasing, point of inflection at origin) with approximate shape through marked points
- **(a) Answer: (-1, -1), (0, 0), (1, 1)**

Question 2

- (a) **B1** translation 2 units right and 1 unit up oe
- **(a) Answer: Translate 2 units to the right and 1 unit up.**
- (b) **M1** apply translation to (2, 8): (2+2, 8+1) or equivalent method
- (b) **A1** (4, 9) cao
- **(b) Answer: (4, 9)**

Question 3

- (a) **B1** vertical asymptote x = 0 stated
- (a) **B1** horizontal asymptote y = 0 stated
- **(a) Answer: Vertical asymptote x = 0. Horizontal asymptote y = 0.**
- (b) **M1** solve 2/x = 1 or equivalent
- (b) **A1** (2, 1) cao
- **(b) Answer: (2, 1)**

Question 4

- (a) **B1** substitute y = 2x into $x^3 - 3x = y$ giving $x^3 - 3x = 2x$ and rearrange to $x^3 - 5x = 0$ oe
- **(a) Answer: $x^3 - 5x = 0$**
- (b) **M1** factor $x^3 - 5x = x(x^2 - 5)$ and find x = 0, $x = \sqrt{5}$, $x = -\sqrt{5}$ (allow correct factorisation shown)
- (b) **M1** find corresponding y by y = 2x or substitution
- (b) **A1** (0, 0), ($\sqrt{5}$, 2sqrt(5)), (-$\sqrt{5}$, -2sqrt(5)) cao
- **(b) Answer: (0, 0), ($\sqrt{5}$, 2 $\sqrt{5}$), (-$\sqrt{5}$, -2 $\sqrt{5}$)**

Question 5

- (a) **B1** vertical asymptote x = -1 stated
- (a) **B1** horizontal asymptote y = 4 stated
- **(a) Answer: Vertical asymptote x = -1. Horizontal asymptote y = 4.**
- (b) **M1** set y = 0 and solve -3/(x+1) + 4 = 0 leading to -3/(x+1) = -4
- (b) **A1** x = -1/4 cao
- **(b) Answer: x = -1/4**

Question 6

- (a) **M1** either show synthetic division by x=1 then x=2 or factor by grouping leading to factors (x-1),(x-2),(x-3)
- (a) **A1** (x - 1)(x - 2)(x - 3) cao
- **(a) Answer: (x - 1)(x - 2)(x - 3)**
- (b) **M1** sketch shows cubic crossing x-axis at three points and general shape correct
- (b) **A1** x-intercepts (1, 0), (2, 0), (3, 0) cao
- **(b) Answer: (1, 0), (2, 0), (3, 0)**

Question 7

- (a) **B1** substitute x = 0 to give y = 0 thereby showing x=0 is a root
- **(a) Answer: Substitute x = 0 gives y = 0, so x = 0 is a root.**
- (b) **M1** factor $y = x^3 - 3x^2$ as $x^2(x - 3)$ or equivalent to find roots
- (b) **M1** identify roots x = 0 (double), x = 3 or explicitly x = 0,0,3
- (b) **M1** work out values e.g. y(-1) = -4, y(0) = 0, y(1) = -2, y(2) = -4, y(3) = 0 and compare neighbouring values around x = 0 and x = 2
- (b) **A1** x = 0 is a local maximum (0 is higher than the neighbouring values -4 and -2) and x = 2 is a local minimum (-4 is lower than the neighbouring values -2 and 0) cao
- **(b) Answer: Other roots: x = 0 (double root) and x = 3. Using y(-1) = -4, y(0) = 0, y(1) = -2, y(2) = -4, y(3) = 0: x = 0 is a local maximum (0 is higher than the values on either side, -4 and -2) and x = 2 is a local minimum (-4 is lower than the values on either side, -2 and 0).**

Question 8

- (a) **M1** state that for x > 0, as x increases, x^2 increases, so $y = 1/x^2$ decreases (support with values, e.g. y(1) = 1, y(2) = 0.25, y(3) = 1/9)
- (a) **A1** conclude y is strictly decreasing throughout x > 0, so it never changes from increasing to decreasing (or vice versa) and cannot have a turning point oe
- **(a) Answer: For x > 0, as x increases x^2 increases so $y = 1/x^2$ strictly decreases throughout; since the graph never stops decreasing, it cannot have a turning point for x > 0.**
- (b) **M1** state that the graph is strictly decreasing for x > 0 (no local max/min)
- (b) **A1** conclude no turning points for x>0 so graph decreases from +infinity towards 0 approaching horizontal asymptote y=0 cao
- **(b) Answer: For x > 0 the graph is strictly decreasing throughout: it falls from +infinity as x->0+ and approaches the horizontal asymptote y = 0 as x->infinity; there are no local maxima or minima for x>0.**