5.12 Solving Simultaneous Equations Graphically: 2-hour Pack

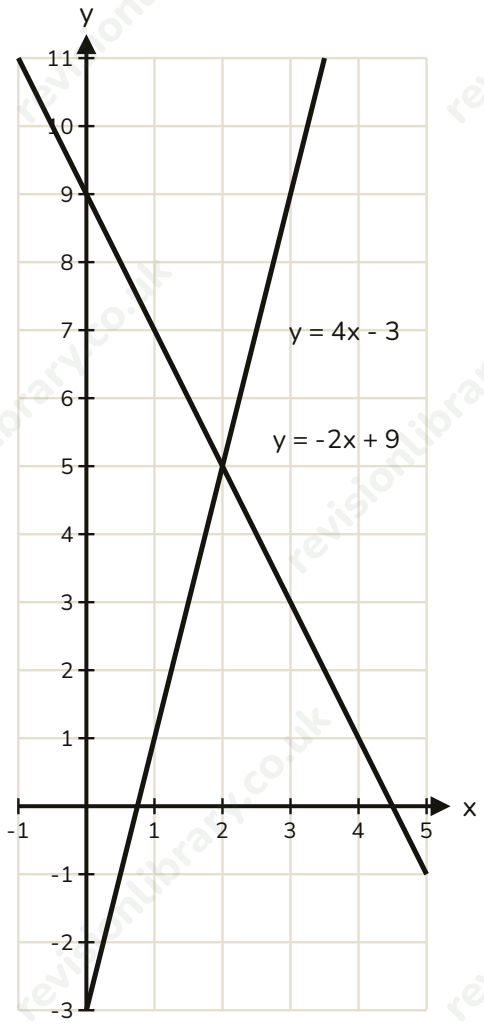
Total Marks

EDEXCEL 1MA1 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The graphs of y = -2x + 9 and y = 4x - 3 are drawn on the grid.
Write down the solution of the simultaneous equations y = -2x + 9 and y = 4x - 3.

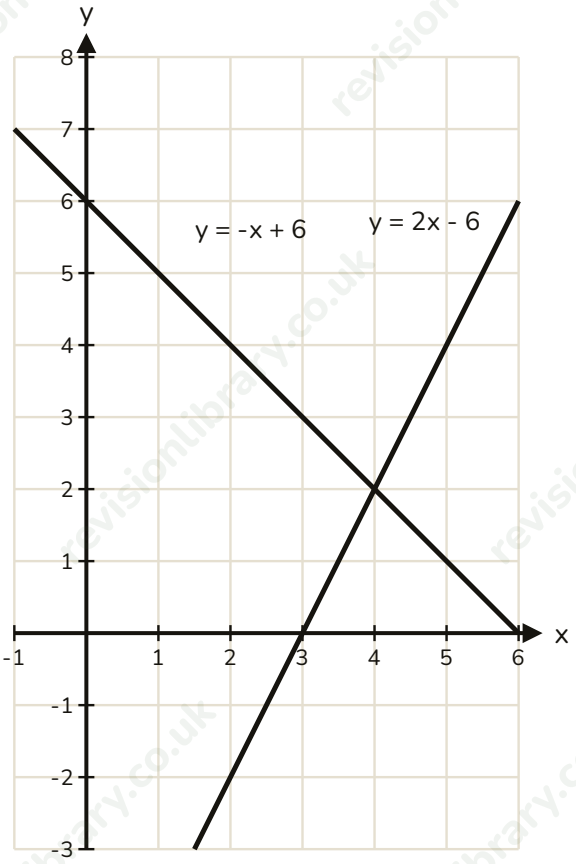


(Total for Question 1 is 2 marks)

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2 The graphs of $y = 2x - 6$ and $y = -x + 6$ are drawn on the grid.

Write down the solution of the simultaneous equations $y = 2x - 6$ and $y = -x + 6$.

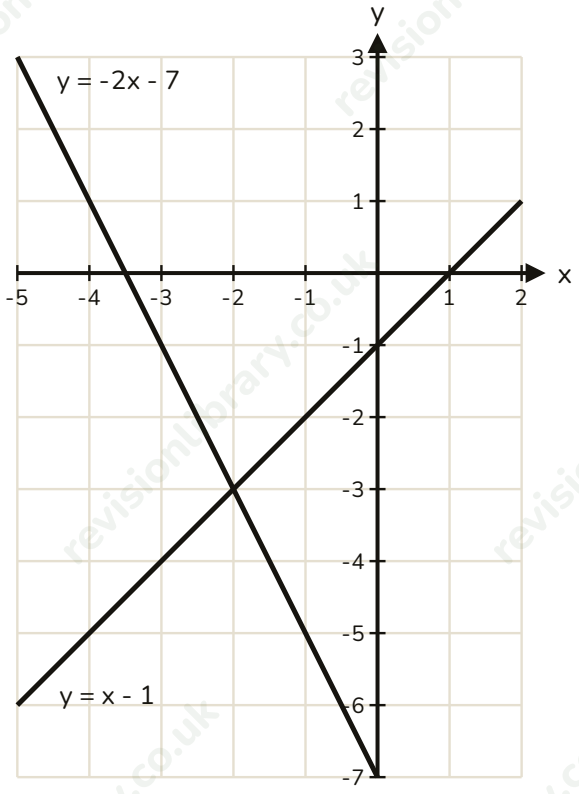


(Total for Question 2 is 2 marks)

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3 The graphs of $y = x - 1$ and $y = -2x - 7$ are drawn on the grid.

Write down the solution of the simultaneous equations $y = x - 1$ and $y = -2x - 7$.

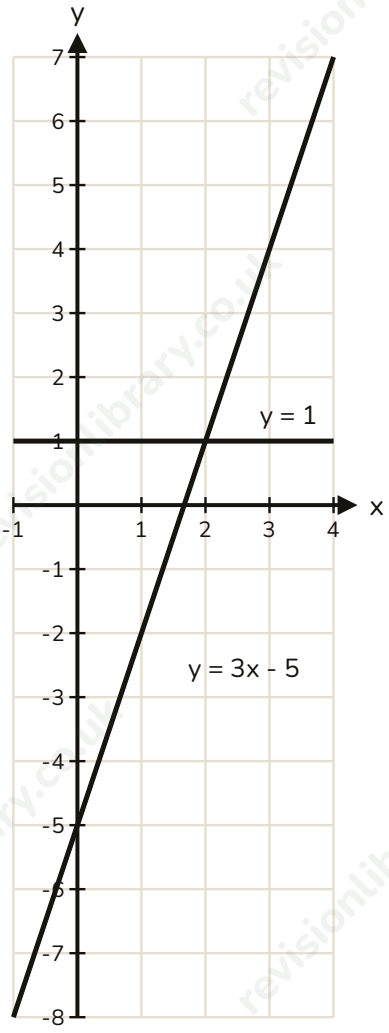


(Total for Question 3 is 2 marks)

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4 The graphs of $y = 1$ and $y = 3x - 5$ are drawn on the grid.

Write down the solution of the simultaneous equations $y = 1$ and $y = 3x - 5$.



(Total for Question 4 is 2 marks)

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5 The straight lines $2x + 2y = 20$ and $y = 3x - 2$ are drawn on the same grid. Draw both lines and use your graph to solve the simultaneous equations. Show how you found two points for each line.

(Total for Question 5 is 4 marks)

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6 The straight lines $2x + 2y = 12$ and $x - 2y = -18$ are drawn on the same grid. Draw both lines and use your graph to solve the simultaneous equations. Show how you found two points for each line.

(Total for Question 6 is 4 marks)

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7 The lines $y = 4x + 2$ and $-4x + y = -4$ are drawn on the same grid. Explain what the graph shows about the solutions of the simultaneous equations.

(Total for Question 7 is 2 marks)

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8 Two plumbers charge as follows. Firm A: a fixed charge of £5 plus £5 per hour. Firm B: a fixed charge of £25 plus £1 per hour. Draw a graph of the cost against the number of hours for each firm on the same axes, and use it to find the number of hours for which the two firms charge the same, and what that charge is.

(Total for Question 8 is 3 marks)

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9 Complete the table of values for y = -3x - 2, for x = -3, -2, 0, 3. Give the four y values in order.

(Total for Question 9 is 2 marks)

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10 Two taxi firms charge as follows. Firm A: a fixed charge of £4 plus £8 per mile. Firm B: a fixed charge of £20 plus £6 per mile. Draw a graph of the cost against the number of miles for each firm on the same axes, and use it to find the number of miles for which the two firms charge the same, and what that charge is.

(Total for Question 10 is 3 marks)

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11 The lines y = 4x + 4 and -4x + y = -3 are drawn on the same grid. Explain what the graph shows about the solutions of the simultaneous equations.

(Total for Question 11 is 2 marks)

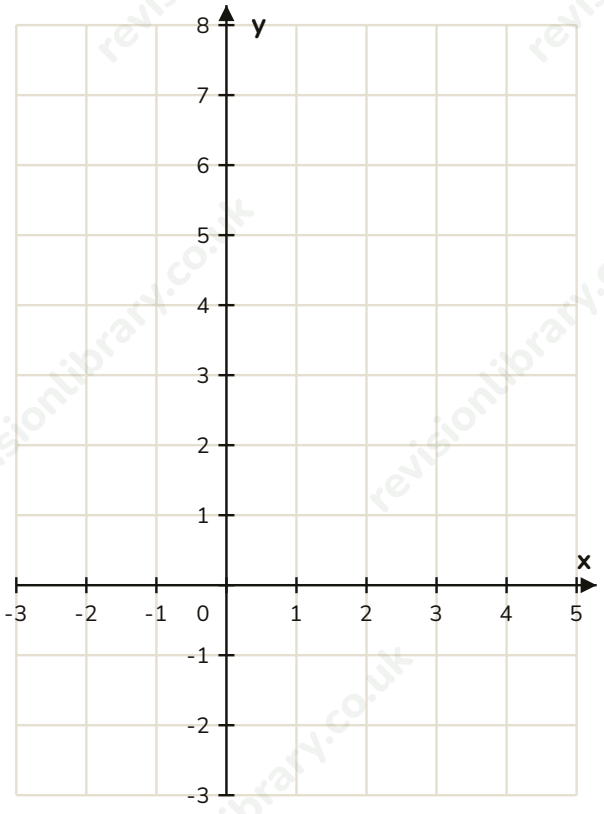
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12 Two van hire firms charge as follows. Firm A: a fixed charge of £8 plus £9 per day. Firm B: a fixed charge of £32 plus £1 per day. Draw a graph of the cost against the number of days for each firm on the same axes, and use it to find the number of days for which the two firms charge the same, and what that charge is.

(Total for Question 12 is 3 marks)

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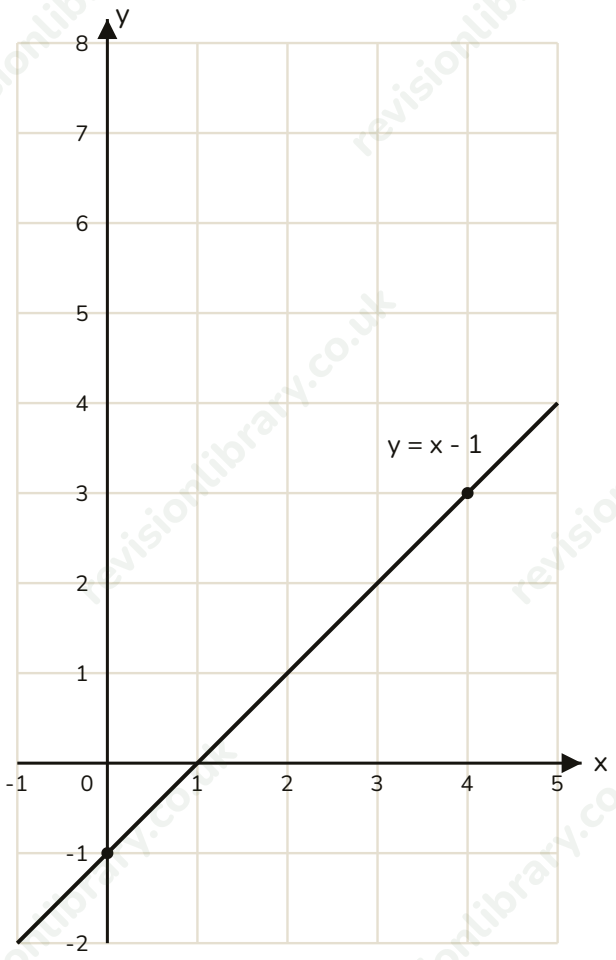
13 By drawing suitable straight lines on the grid, solve the simultaneous equations $y = x + 3$ and $y = 2x + 1$.



(Total for Question 13 is 4 marks)

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14 This question has two parts.



(a) Rearrange $2x + y = 8$ to make y the subject. **(1)**

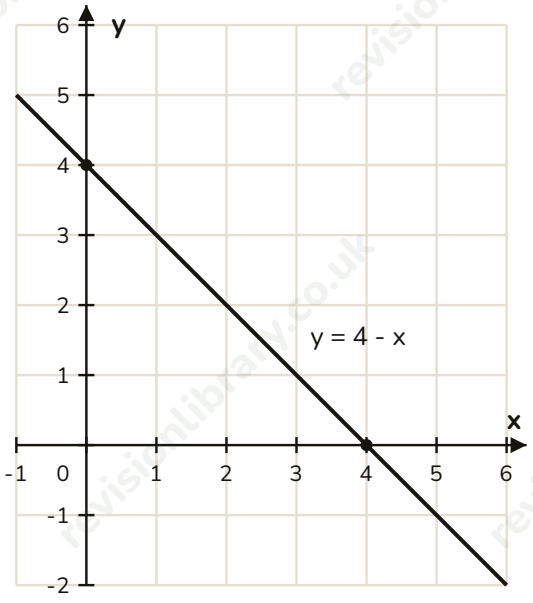
(b) The line $y = x - 1$ is already drawn on the grid. By completing a table of values and drawing the graph of $y = 8 - 2x$ on the same grid, solve the simultaneous equations $2x + y = 8$ and $y = x - 1$. **(4)**

.....

(Total for Question 14 is 5 marks)

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15 The line y = 4 - x is already drawn on the grid. By drawing the graph of y = 2x - 5 on the same grid, solve the simultaneous equations y = 4 - x and y = 2x - 5.



(Total for Question 15 is 4 marks)

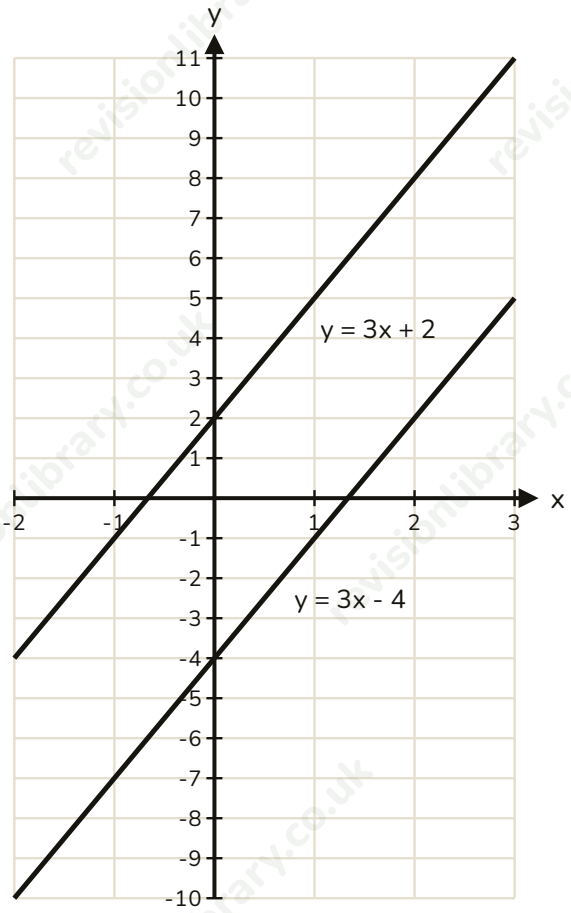
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16 Show that the point (3, -1) is the solution to the simultaneous equations y = 2x - 7 and y = -3x + 8.

(Total for Question 16 is 2 marks)

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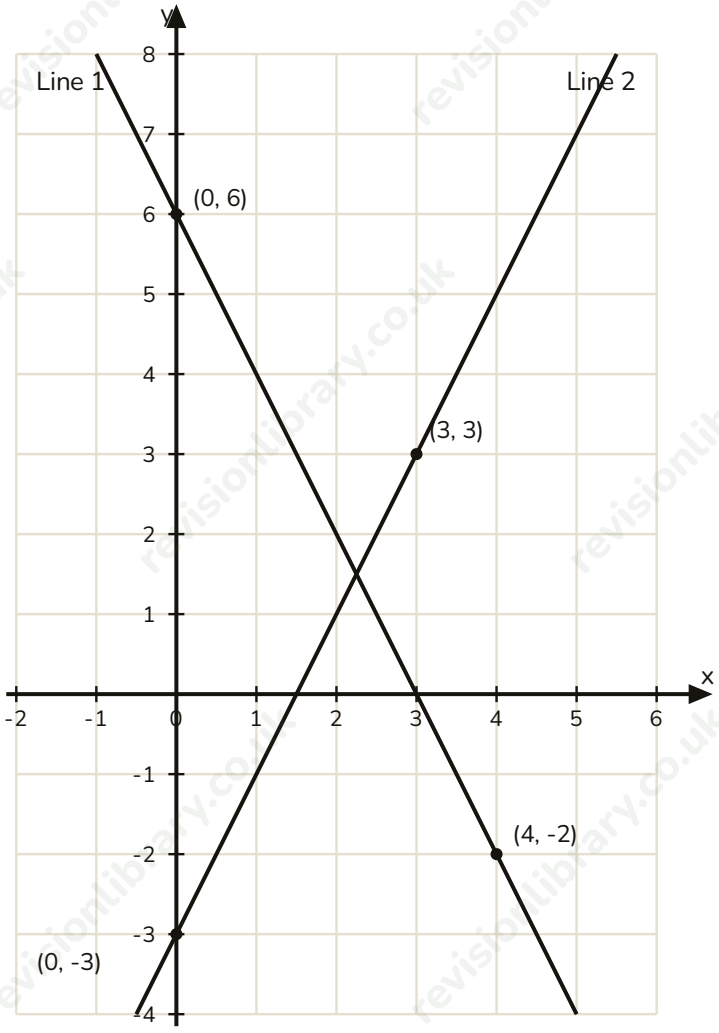
17 The lines y = 3x + 2 and y = 3x - 4 are shown on the grid. They never meet.
What does this tell you about the solution of the simultaneous equations y = 3x + 2 and y = 3x - 4?



(Total for Question 17 is 1 mark)

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18 The diagram shows two straight lines drawn on a grid. Line 1 passes through (0, 6) and (4, -2). Line 2 passes through (0, -3) and (3, 3).



(a) Find the equation of Line 1 in the form y = mx + c. **(2)**

.....

(b) Find the equation of Line 2 in the form y = mx + c. **(2)**

.....

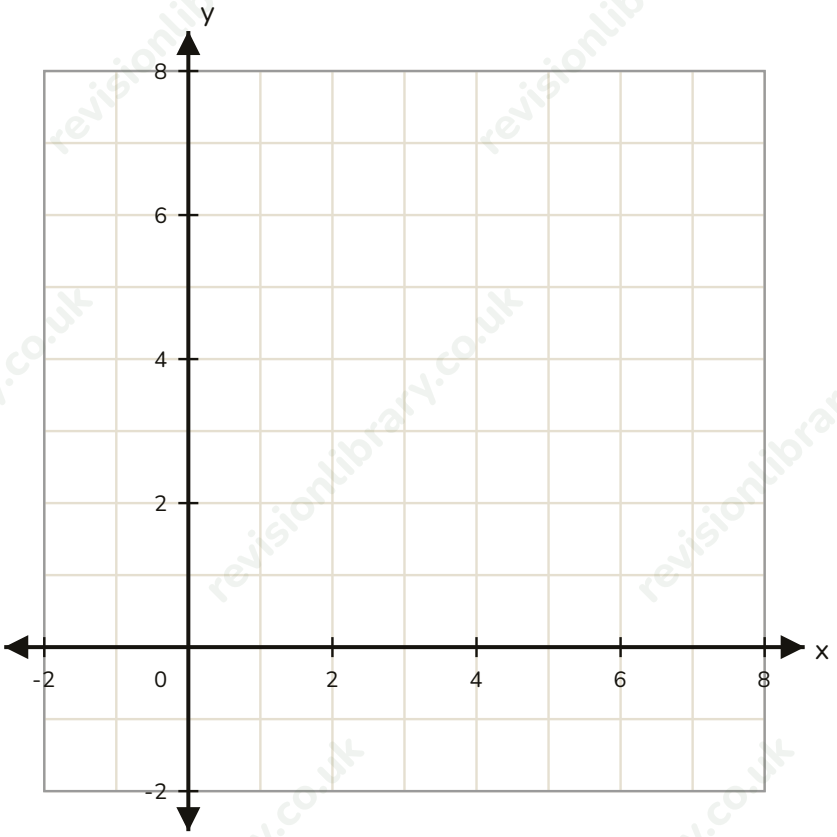
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(c) Hence find the coordinates of the point of intersection of the two lines, showing your method. **(3)**

(Total for Question 18 is 7 marks)

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19 This question has three parts.



(a) Rearrange $3x - 2y = 6$ to make y the subject. **(1)**

(b) Rearrange $x + y = 7$ to make y the subject. **(1)**

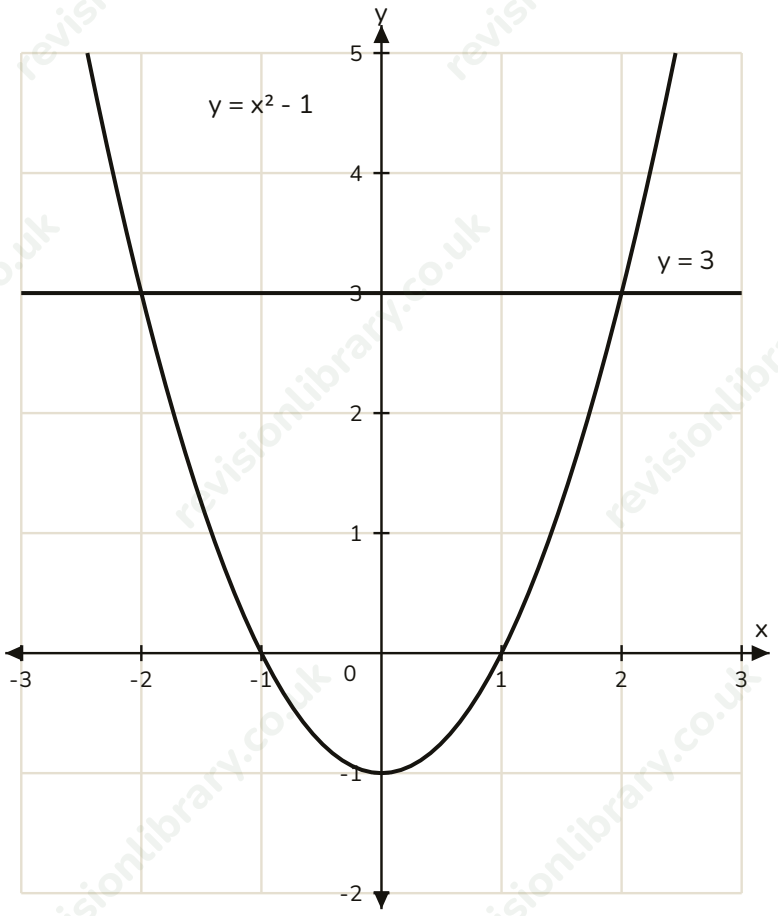
(c) By drawing suitable graphs on the grid, solve the simultaneous equations $3x - 2y = 6$ and $x + y = 7$. **(4)**

.....

(Total for Question 19 is 6 marks)

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20 The graphs of $y = x^2 - 1$ and $y = 3$ are drawn on the same grid. How many points of intersection are there, and what does this tell you about the number of solutions to the equation $x^2 - 1 = 3$?



- ☐ A) 0 intersections, no solutions
- ☐ B) 1 intersection, 1 solution
- ☐ C) 2 intersections, 2 solutions
- ☐ D) 3 intersections, 3 solutions

(Total for Question 20 is 1 mark)

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21 The graph of $y = kx + 2$ meets the curve $y = 3 - x^2$ at exactly one point. Find the value(s) of k.

(Total for Question 21 is 4 marks)

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22 A straight line passes through the points where $y = x^2 - 5x + 4$ crosses the x-axis. Find the equation of this straight line and hence find its intersections with the curve $y = x^2 - 5x + 4$ again, confirming consistency.

(Total for Question 22 is 4 marks)

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23 The lines $y = kx + 4$ and $y = 3x - 2$ intersect at the point where $x = 3$.

(a) The lines $y = kx + 4$ and $y = 3x - 2$ intersect at the point where $x = 3$. Find the y-coordinate of the point of intersection. **(1)**

.....

(b) Hence find the value of k. **(2)**

.....

(Total for Question 23 is 3 marks)

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24 Show that the graphs of $y = 2x + 1$ and $y = -x^2 + 5x - 1$ intersect at $x = 1$ and $x = 2$. Then find the y-coordinates of the intersection points and give the coordinates.

(Total for Question 24 is 5 marks)

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25 Consider the pair of equations $y = 0.2x^2 + x - 1$ and $y = 0.8x + 2$. Use algebra to form the equation whose roots are the x-coordinates of the intersections. Then find those x-coordinates to 2 decimal places and give the intersection points.

(Total for Question 25 is 3 marks)

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26 The straight line L has equation $y = mx + 3$ and is tangent to the curve $y = x^2 - 2x + 7$ at $x = a$. Using the condition that the line and curve have equal gradients at the point of tangency and that they meet at $x = a$, express m in terms of a, and find the possible value(s) of a. Give the coordinates of the tangency point when $a = 2$.

(Total for Question 26 is 6 marks)

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27 Two lines have equations $y = kx + 2$ and $y = 3x + 2$, where k is a constant and k is not equal to 3.

(a) Show that the point (0, 2) lies on both lines for every value of k. **(2)**

(b) Explain why (0, 2) must be the only point of intersection of the two lines when k is not equal to 3. **(2)**

(Total for Question 27 is 4 marks)

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Mark scheme · 5.12 Solving Simultaneous Equations Graphically: 2-hour Pack

Question 1

x = 2	**B1**
y = 5 (oe as a coordinate pair)	**B1**

Answer: x = 2, y = 5

Question 2

x = 4	**B1**
y = 2 (oe as a coordinate pair)	**B1**

Answer: x = 4, y = 2

Question 3

x = -2	**B1**
y = -3 (oe as a coordinate pair)	**B1**

Answer: x = -2, y = -3

Question 4

x = 2	**B1**
y = 1 (oe as a coordinate pair)	**B1**

Answer: x = 2, y = 1

Question 5

table or two points on first line found correctly (e.g. intercepts)	**M1**
two points on second line found correctly	**M1**
both lines drawn and intersection identified	**M1**
x = 3, y = 7 cao	**A1**

Answer: x = 3, y = 7

Question 6

table or two points on first line found correctly (e.g. intercepts)	**M1**
two points on second line found correctly	**M1**
both lines drawn and intersection identified	**M1**
x = -2, y = 8 cao	**A1**

Answer: x = -2, y = 8

Question 7

lines have the same gradient	**B1**
lines never meet so there is no solution	**B1**

Answer: No solution because the lines are parallel

Question 8

line for Firm A drawn or equation C = 5x + 5	**M1**
line for Firm B drawn or equation C = x + 25	**M1**
5 hours, £30 cao	**A1**

Answer: 5 hours, £30

Question 9

substitutes at least two x values correctly	**M1**
7, 4, -2, -11 cao	**A1**

Answer: 7, 4, -2, -11

Question 10

line for Firm A drawn or equation C = 8x + 4	**M1**
line for Firm B drawn or equation C = 6x + 20	**M1**
8 miles, £68 cao	**A1**

Answer: 8 miles, £68

Question 11

lines have the same gradient	**B1**
lines never meet so there is no solution	**B1**

Answer: No solution because the lines are parallel

Question 12

line for Firm A drawn or equation C = 9x + 8	**M1**
line for Firm B drawn or equation C = x + 32	**M1**
3 days, £35 cao	**A1**

Answer: 3 days, £35

Question 13

draws y = x + 3 correctly through at least 3 correct points, e.g. (0,3), (1,4), (-2,1)	**M1**
draws y = 2x + 1 correctly through at least 3 correct points, e.g. (0,1), (1,3), (2,5)	**M1**
identifies intersection point at (2,5)	**A1**
states solution x = 2, y = 5 (ft from their graph)	**A1**

Answer: x = 2, y = 5

Question 14

(a) y = 8 - 2x (oe)	**B1**

(a) Answer: y = 8 - 2x

(b) table of at least 3 correct values for y = 8 - 2x, e.g. x=0,y=8; x=2,y=4; x=4,y=0	**M1**
(b) correct line y = 8 - 2x drawn, ft from table	**M1**
(b) identifies intersection at (3,2)	**A1**
(b) states solution x = 3, y = 2 (ft)	**A1**

(b) Answer: x = 3, y = 2

Question 15

at least 3 correct points for y = 2x - 5, e.g. (0,-5), (2,-1), (3,1)	**M1**
correct line y = 2x - 5 drawn	**A1**
identifies intersection at (3,1)	**B1**
states solution x = 3, y = 1 (ft)	**B1**

Answer: x = 3, y = 1

Question 16

substitutes x = 3 into both equations	**M1**
shows y = -1 from both equations, cso, so (3,-1) satisfies both equations	**A1**

Answer: y = 2(3) - 7 = -1 and y = -3(3) + 8 = -1, both give y = -1, so (3,-1) satisfies both equations

Question 17

there is no solution, because the lines are parallel (same gradient, different y-intercepts) so they never intersect	**B1**

Answer: No solution - the lines are parallel and never meet

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Question 18

(a) gradient = (-2 - 6) / (4 - 0) = -2 **M1**

(a) y = -2x + 6 (oe) **A1**

(a) Answer: y = -2x + 6

(b) gradient = (3 - (-3)) / (3 - 0) = 2 **M1**

(b) y = 2x - 3 (oe) **A1**

(b) Answer: y = 2x - 3

(c) sets -2x + 6 = 2x - 3 **M1**

(c) correctly solves for x: 9 = 4x, x = 2.25 **M1**

(c) y = 1.5, coordinates (2.25, 1.5) cao **A1**

(c) Answer: (2.25, 1.5)

Question 19

(a) y = 1.5x - 3 (oe, e.g. y = (3x - 6) / 2) **B1**

(a) Answer: y = 1.5x - 3

(b) y = 7 - x **B1**

(b) Answer: y = 7 - x

(c) draws y = 1.5x - 3 correctly, ft from (a), e.g. through (0,-3), (2,0), (4,3) **M1**

(c) draws y = 7 - x correctly, ft from (b), e.g. through (0,7), (4,3), (7,0) **M1**

(c) identifies intersection at (4,3) **A1**

(c) states solution x = 4, y = 3 (ft) **A1**

(c) Answer: x = 4, y = 3

Question 20

C **B1**

Answer: C) 2 intersections, 2 solutions

Question 21

set $kx + 2 = 3 - x^2$ to form $x^2 + kx - 1 = 0$ and state condition for one root is discriminant = 0 **M1**

compute discriminant: $k^2 - 4(1)(-1) = k^2 + 4$ **M1**

set $k^2 + 4 = 0$ and conclude no real k makes discriminant zero, or recognise discriminant always positive **M1**

no real value of k gives exactly one real intersection; always two real intersections for real k cao **A1**

Answer: No real value of k. The discriminant $k^2 + 4$ is always positive for real k, so the quadratic has two distinct real roots for every real k.

Question 22

find x-intercepts of curve: solve $x^2 - 5x + 4 = 0$ giving x = 1 and x = 4 **M1**

find equation of line through (1,0) and (4,0): recognise it is the x-axis y = 0 **M1**

substitute y = 0 into curve to verify intersections at x = 1 and x = 4 **M1**

equation y = 0 and intersections x = 1, 4 cao **A1**

Answer: Equation: y = 0. Intersections with the curve occur at x = 1 and x = 4 (points (1,0) and (4,0)).

Question 23

(a) y = 7 **B1**

(a) Answer: y = 7

(b) substitutes the point (3, 7) into y = kx + 4 **M1**

(b) k = 1 (ft from (a)) **A1**

(b) Answer: k = 1

Question 24

equate $2x + 1 = -x^2 + 5x - 1$ to get $x^2 - 3x + 2 = 0$ and show factor or algebraic manipulation leading to (x-1)(x-2)=0 or show substitution gives zero **M1**

obtain x = 1 and x = 2 (show roots) oe **M1**

substitute x = 1 into y = 2x + 1 to get y = 3, and x = 2 gives y = 5 **M1**

coordinates (1, 3) and (2, 5) cao **A1**

explicit statement 'hence' linking substitution to intersection coordinates **B1**

Answer: Intersection points (1, 3) and (2, 5).

Question 25

equate $0.2x^2 + x - 1 = 0.8x + 2$ to form $0.2x^2 + 0.2x - 3 = 0$ or multiply to $x^2 + x - 15 = 0$ **M1**

use quadratic formula to find $x = [-1 \pm \sqrt{1 + 60}]/2 = [-1 \pm \sqrt{61}]/2$ **M1**

x approx (-1 + 7.8102)/2 = 3.4051 -> 3.41 and x approx (-1 - 7.8102)/2 = -4.4051 -> -4.41; intersection points (3.41, 4.72) and (-4.41, -1.52) cao (accept 4.73 and -1.53 if the rounded x-values are used) **A1**

Answer: x = (-1 ± $\sqrt{61}$)/2 approx 3.41 and -4.41. Intersection points approx (3.41, 0.8*3.4051 + 2 = 4.72) and (-4.41, 0.8* (-4.4051) + 2 = -1.52) to 2 dp.

Question 26

state gradient of curve dy/dx = 2x - 2 and set m = 2a - 2 for tangent at x = a **M1**

state point of contact satisfies $a^2 - 2a + 7 = m\,a + 3$ **M1**

substitute m = 2a - 2 into equation to get $a^2 - 2a + 7 = (2a - 2)a + 3 = 2a^2 - 2a + 3$ **M1**

rearrange correctly to $-a^2 + 4 = 0$, i.e. $a^2 = 4$, giving a = 2 or a = -2 **M1**

state a = 2 gives m = 2(2) - 2 = 2, and a = -2 gives m = 2(-2) - 2 = -6 cao **A1**

for given a = 2, compute coordinates: $y = 2^2 - 2(2) + 7 = 7$, so tangency point (2, 7), with working shown cao **A1**

Answer: m = 2a - 2; tangency occurs when a = 2 (m = 2) or a = -2 (m = -6). For a = 2 the tangency point is (2, 7).

Question 27

(a) substitutes x = 0 into both y = kx + 2 and y = 3x + 2 **M1**

(a) shows both give y = 2 regardless of the value of k, cso, so (0,2) satisfies both equations **A1**

(a) Answer: y = k(0) + 2 = 2 and y = 3(0) + 2 = 2, both give y = 2 for every k, so (0,2) lies on both lines

(b) since k is not equal to 3, the two lines have different gradients (k and 3), so they are not parallel and not identical **B1**

(b) two distinct straight lines with different gradients intersect at exactly one point, and (a) shows this point is (0,2), so it must be the unique intersection **B1**

(b) Answer: Because k does not equal 3 the lines have different gradients, so they are not parallel; two non-parallel lines meet at exactly one point, and that point must be (0,2)

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