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HIGHER

5.12H Solving Simultaneous Equations Graphically: Higher Tier Practice

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 75 minutes

Total Marks

Name: ______________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 Two straight lines are given by $y = 2x - 3$ and $y = -x + 4$. Work out the coordinates of their intersection point.

(Total for Question 1 is 2 marks)

2 A line with equation $y = 0.5x + 1$ and a parabola with equation $y = x^2 - 2x + 2$ are drawn on the same axes. Work out the x-coordinates of their points of intersection.

(Total for Question 2 is 3 marks)

3 Sketch on the same axes the graphs of $y = -x + 6$ and $y = x^2 - 4x + 3$ and use your sketch to estimate the x-coordinates of their intersections. Give your answers to 1 decimal place.

(Total for Question 3 is 3 marks)

4 The graph of $y = kx + 2$ meets the curve $y = 3 - x^2$ at exactly one point. Find the value(s) of k.

(Total for Question 4 is 4 marks)

5 A straight line passes through the points where $y = x^2 - 5x + 4$ crosses the x-axis. Find the equation of this straight line and hence find its intersections with the curve $y = x^2 - 5x + 4$ again, confirming consistency.

(Total for Question 5 is 4 marks)

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6 Show that the graphs of $y = 2x + 1$ and $y = -x^2 + 5x - 1$ intersect at $x = 1$ and $x = 2$. Then find the y-coordinates of the intersection points and give the coordinates.

(Total for Question 6 is 5 marks)

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7 The straight line L has equation $y = mx + 3$ and is tangent to the curve $y = x^2 - 2x + 7$ at $x = a$. Using the condition that the line and curve have equal gradients at the point of tangency and that they meet at $x = a$, express m in terms of a, and find the possible value(s) of a. Give the coordinates of the tangency point when $a = 2$.

(Total for Question 7 is 6 marks)

8 Consider the pair of equations $y = 0.2x^2 + x - 1$ and $y = 0.8x + 2$. Use algebra to form the equation whose roots are the x-coordinates of the intersections. Then find those x-coordinates to 2 decimal places and give the intersection points.

(Total for Question 8 is 3 marks)

Mark scheme · 5.12H Solving Simultaneous Equations Graphically: Higher Tier Practice

Question 1

- **M1** substitute or set $2x - 3 = -x + 4$ and solve for x
- **A1** $x = 7/3$, $y = 5/3$ cao
- **Answer:(7/3, 5/3)**
  (Approximately (2.333, 1.667))

Question 2

- **M1** equate $x^2 - 2x + 2 = 0.5x + 1$ to form quadratic $x^2 - 2.5x + 1 = 0$
- **M1** use quadratic formula or complete the square to find x
- **A1** $x = (2.5 \pm \sqrt{2.25})/2 = (2.5 \pm 1.5)/2$ so $x = 2$ or $x = 0.5$ cao
- **Answer:x = 2 and x = 0.5**
  (Points of intersection at x = 2 and x = 1/2)

Question 3

- **M1** sketch showing both graphs with at least one intersection roughly located and correct axes scale
- **M1** identify two intersection x-values from sketch within reasonable accuracy
- **A1** answers x approx -0.8 and x approx 3.8 to 1 dp (or equivalent to 1 dp) cao
- **Answer: x = -0.8 and x = 3.8 (to 1 decimal place)**

Question 4

- **M1** set $kx + 2 = 3 - x^2$ to form $x^2 + kx - 1 = 0$ and state condition for one root is discriminant = 0
- **M1** compute discriminant: $k^2 - 4(1)(-1) = k^2 + 4$
- **M1** set $k^2 + 4 = 0$ and conclude no real k makes discriminant zero, or recognise discriminant always positive
- **A1** no real value of k gives exactly one real intersection; always two real intersections for real k cao
- **Answer: No real value of k. The discriminant $k^2 + 4$ is always positive for real k, so the quadratic has two distinct real roots for every real k.**

Question 5

- **M1** find x-intercepts of curve: solve $x^2 - 5x + 4 = 0$ giving $x = 1$ and $x = 4$
- **M1** find equation of line through (1,0) and (4,0): recognise it is the x-axis $y = 0$
- **M1** substitute $y = 0$ into curve to verify intersections at $x = 1$ and $x = 4$
- **A1** equation $y = 0$ and intersections $x = 1, 4$ cao
- **Answer: Equation: y = 0. Intersections with the curve occur at x = 1 and x = 4 (points (1,0) and (4,0)).**

Question 6

- **M1** equate $2x + 1 = -x^2 + 5x - 1$ to get $x^2 - 3x + 2 = 0$ and show factor or algebraic manipulation leading to $(x-1)(x-2)=0$ or show substitution gives zero
- **M1** obtain $x = 1$ and $x = 2$ (show roots) oe
- **M1** substitute $x = 1$ into $y = 2x + 1$ to get $y = 3$, and $x = 2$ gives $y = 5$
- **A1** coordinates (1, 3) and (2, 5) cao
- **B1** explicit statement 'hence' linking substitution to intersection coordinates
- **Answer: Intersection points (1, 3) and (2, 5).**

Question 7

- **M1** state gradient of curve $dy/dx = 2x - 2$ and set $m = 2a - 2$ for tangent at $x = a$
- **M1** state point of contact satisfies $a^2 - 2a + 7 = m a + 3$
- **M1** substitute $m = 2a - 2$ into equation to get $a^2 - 2a + 7 = (2a - 2)a + 3 = 2a^2 - 2a + 3$
- **M1** rearrange correctly to $-a^2 + 4 = 0$, i.e. $a^2 = 4$, giving $a = 2$ or $a = -2$
- **A1** state $a = 2$ gives $m = 2(2) - 2 = 2$, and $a = -2$ gives $m = 2(-2) - 2 = -6$ cao
- **A1** for given $a = 2$, compute coordinates: $y = 2^2 - 2(2) + 7 = 7$, so tangency point (2, 7), with working shown cao
- **Answer: $m = 2a - 2$; tangency occurs when $a = 2$ ($m = 2$) or $a = -2$ ($m = -6$). For $a = 2$ the tangency point is (2, 7).**

Question 8

- **M1** equate $0.2x^2 + x - 1 = 0.8x + 2$ to form $0.2x^2 + 0.2x - 3 = 0$ or multiply to $x^2 + x - 15 = 0$
- **M1** use quadratic formula to find $x = [-1 \pm \sqrt{1 + 60}]/2 = [-1 \pm \sqrt{61}]/2$
- **A1** x approx $(-1 + 7.8102)/2 = 3.4051$ -> 3.41 and x approx $(-1 - 7.8102)/2 = -4.4051$ -> -4.41; intersection points (3.41, y) and (-4.41, y) cao
- **Answer: $x = (-1 \pm \sqrt{61})/2$ approx 3.41 and -4.41. Intersection points approx (3.41, 0.8*3.41 + 2 = 4.73) and (-4.41, 0.8*(-4.41) + 2 = -1.53) to 2 dp.**