



## 5.20H Exact trig values: Higher Tier Practice

EDEXCEL 1MA1 · Calculator allowed · about 75 minutes

Total Marks

Name: \_\_\_\_\_ Date: \_\_\_\_ / \_\_\_\_ / \_\_\_\_

**Answer ALL questions. Show all your working.**

**1** Write down the exact values of each of the following.

a)  $\sin 30$  degrees

b)  $\cos 45$  degrees

c)  $\tan 60$  degrees

(a) Write down  $\sin 30$  degrees.

(1)

(b) Write down  $\cos 45$  degrees.

(1)

(c) Write down  $\tan 60$  degrees.

(1)

(Total for Question 1 is 3 marks)

**2** Show that  $\sin 15$  degrees  $= (\sqrt{6} - \sqrt{2}) / 4$ . You may use exact values for  $\sin 30$ ,  $\cos 30$ ,  $\sin 45$  and  $\cos 45$ .

(main) Show that  $\sin 15$  degrees  $= (\sqrt{6} - \sqrt{2}) / 4$ .

(3)

(Total for Question 2 is 3 marks)

3 Show that  $\tan 15^\circ = 2 - \sqrt{3}$ . You may use exact values or angle formulae.

**(main)** Show that  $\tan 15^\circ = 2 - \sqrt{3}$ .

(3)

(Total for Question 3 is 3 marks)

---

4 Simplify the following expression to an exact surd form, showing working.

$$(1 - \cos 60^\circ) / \sin 30^\circ$$

**(main)** Simplify  $(1 - \cos 60^\circ) / \sin 30^\circ$  to an exact value.

(3)

(Total for Question 4 is 3 marks)

---

5 Solve for  $\theta$ , giving exact answers in degrees, in the interval  $0 \leq \theta < 180^\circ$ .

$$\sin \theta = \sqrt{3}/2$$

**(main)** Solve  $\sin \theta = \sqrt{3}/2$  for  $0 \leq \theta < 180^\circ$ .

(2)

.....  
(Total for Question 5 is 2 marks)

---

6 Write down the exact value of  $\cos 75^\circ$ . Give a brief reason.

**(main)** Write down  $\cos 75^\circ$  and give a reason.

(2)

.....  
(Total for Question 6 is 2 marks)

7 Show that  $\tan 75^\circ = 2 + \sqrt{3}$ . You may use exact values or formulae.

**(main)** Show that  $\tan 75^\circ = 2 + \sqrt{3}$ .

(3)

(Total for Question 7 is 3 marks)

8 In triangle ABC, angle A = 45 degrees, angle B = 75 degrees and angle C = 60 degrees. The side AB = 1 (the side opposite angle C). Use the sine rule to find the exact lengths of BC and AC, giving answers in simplified surd form.

**(main)** Use the sine rule to find BC and AC, exact surd form, given AB = 1 and angles A = 45, B = 75, C = 60.

(4)

(Total for Question 8 is 4 marks)

9 Show that  $\sin 18^\circ = (\sqrt{5} - 1) / 4$ .

You may use algebraic methods or known exact cosine values; show all steps clearly.

**(main)** Derive  $\sin 18^\circ = (\sqrt{5} - 1) / 4$ , showing working.

(8)

(Total for Question 9 is 8 marks)

**10** Solve the equation  $\tan 2\theta = \sqrt{3}$  for  $0 \leq \theta < 360$  degrees. Give all solutions in degrees and show your working clearly.

**(main)** Solve  $\tan 2\theta = \sqrt{3}$  for  $0 \leq \theta < 360$  degrees, giving all solutions. **(9)**

---

**(Total for Question 10 is 9 marks)**

## Mark scheme · 5.20H Exact trig values: Higher Tier Practice

---

### Question 1

- (a) **B1**  $1/2$  cao
- (a) **Answer:**  $1/2$
- (b) **B1**  $\sqrt{2}/2$  cao
- (b) **Answer:**  $\sqrt{2}/2$
- (c) **B1**  $\sqrt{3}$  cao
- (c) **Answer:**  $\sqrt{3}$

### Question 2

- (main) **M1** use sine difference formula  $\sin(45 - 30) = \sin 45 \cos 30 - \cos 45 \sin 30$  or equivalent
- (main) **M1** substitute exact values:  $(\sqrt{2}/2)(\sqrt{3}/2) - (\sqrt{2}/2)(1/2)$  leading to  $(\sqrt{6} - \sqrt{2})/4$
- (main) **A1**  $(\sqrt{6} - \sqrt{2})/4$  cao, cso
- (main) **Answer:**  $(\sqrt{6} - \sqrt{2})/4$

### Question 3

- (main) **M1** use  $\tan(45 - 30) = (\tan 45 - \tan 30) / (1 + \tan 45 \tan 30)$  or equivalent
- (main) **M1** substitute  $\tan 45 = 1$  and  $\tan 30 = 1/\sqrt{3}$  and simplify to  $(\sqrt{3} - 1)/(\sqrt{3} + 1)$  or equivalent
- (main) **A1**  $2 - \sqrt{3}$  cao
- (main) **Answer:**  $2 - \sqrt{3}$

### Question 4

- (main) **M1** substitute  $\cos 60 = 1/2$  and  $\sin 30 = 1/2$  or show equivalent method
- (main) **M1** compute numerator  $1 - 1/2 = 1/2$  and divide by  $1/2$
- (main) **A1**  $1$  cao
- (main) **Answer:**  $1$

### Question 5

- (main) **M1** identify principal angle  $60$  degrees and use symmetry to find second solution  $120$  degrees
- (main) **A1**  $\theta = 60$  degrees and  $120$  degrees cao
- (main) **Answer:**  $\theta = 60$  degrees or  $120$  degrees

### Question 6

- (main) **B1**  $\cos 75 = \cos(45 + 30)$  or note  $\cos 75 = \sin 15$  oe
- (main) **A1**  $\cos 75 = (\sqrt{6} - \sqrt{2})/4$  cao
- (main) **Answer:**  $\cos 75 \text{ degrees} = (\sqrt{6} - \sqrt{2})/4$

### Question 7

- (main) **M1** use  $\tan(45 + 30) = (\tan 45 + \tan 30) / (1 + \tan 45 \tan 30)$  or equivalent
- (main) **M1** substitute  $\tan 45 = 1$  and  $\tan 30 = 1/\sqrt{3}$  and simplify to  $(\sqrt{3} + 1)/(\sqrt{3} - 1)$  or equivalent
- (main) **A1**  $2 + \sqrt{3}$  cao
- (main) **Answer:**  $2 + \sqrt{3}$

### Question 8

- (main) **M1** use sine rule:  $a/\sin A = b/\sin B = c/\sin C$  and substitute  $c = 1$ ,  $\sin 60 = \sqrt{3}/2$
- (main) **M1** find  $BC = a = \sin 45 / \sin 60$  and simplify to  $\sqrt{6}/3$  or equivalent
- (main) **M1** find  $AC = b = \sin 75 / \sin 60$  and simplify to an exact surd
- (main) **A1**  $BC = \sqrt{6}/3$  and  $AC = (3\sqrt{2} + \sqrt{6})/6$  cao
- (main) **Answer:**  $BC = \sqrt{6}/3$ ,  $AC = (3\sqrt{2} + \sqrt{6})/6$

### Question 9

- (main) **M1** use a suitable exact relation such as  $\cos 36 = (\sqrt{5} + 1)/4$  or derive a polynomial for  $\cos 36$  using double-angle or pentagon symmetry
- (main) **M1** use relationship  $\sin 18 = \sqrt{(1 - \cos 36)/2}$  or equivalent identity
- (main) **M1** substitute  $\cos 36 = (\sqrt{5} + 1)/4$  and simplify under the square root
- (main) **M1** simplify to  $(\sqrt{5} - 1)/4$  as positive root (since  $\sin 18$  is positive)
- (main) **M1** give algebraic justification for selecting the positive root (angle in first quadrant)
- (main) **A1**  $\sin 18 = (\sqrt{5} - 1)/4$  cao, cso
- (main) **M1** alternative valid route: start from 5-fold angle equation for sine/cosine and factor to get the same value
- (main) **A1** final exact form shown clearly
- **(main) Answer:  $\sin 18 \text{ degrees} = (\sqrt{5} - 1)/4$**

### Question 10

- (main) **M1** recognise general solution for  $\tan x = \sqrt{3}$  is  $x = 60 + k \cdot 180$  degrees
- (main) **M1** set  $2\theta = 60 + k \cdot 180$  and solve for  $\theta$  giving  $\theta = 30 + k \cdot 90$  degrees
- (main) **M1** find integer  $k$  values that give  $\theta$  in  $0 \leq \theta < 360$  and list candidates
- (main) **M1** substitute  $k = 0, 1, 2, 3$  to produce  $\theta = 30, 120, 210, 300$  degrees
- (main) **M1** check that each value is distinct and within the interval
- (main) **A1** state final set  $\theta = 30, 120, 210, 300$  degrees cao
- (main) **M1** optional check: show  $\tan 2\theta$  equals  $\sqrt{3}$  for each listed  $\theta$  (e.g.  $2 \cdot 30 = 60$ ,  $\tan 60 = \sqrt{3}$ ) ft
- (main) **M1** note that no other  $k$  produce values in the interval
- (main) **A1** complete solution set given
- **(main) Answer:  $\theta = 30 \text{ degrees}, 120 \text{ degrees}, 210 \text{ degrees}, 300 \text{ degrees}$**