5.23H Venn Diagrams: Higher Tier Practice

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 75 minutes

Total Marks

Name: ______________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 A class of 30 students were surveyed about whether they play chess or football. 18 play chess, 14 play football and 6 play both. Complete a Venn diagram and find how many students play neither.

(Total for Question 1 is 2 marks)

2 In a survey of 120 shoppers, 50 said they buy apples, 70 buy bananas and 40 buy cherries. 30 buy both apples and bananas, 20 buy both bananas and cherries, 15 buy both apples and cherries. Exactly 10 buy all three. Draw a three-set Venn and find the number of shoppers who buy none of these fruits.

(Total for Question 2 is 4 marks)

3 In a town, 400 people were asked if they recycle paper (P) and plastic (L). 260 recycle paper, 180 recycle plastic and 120 recycle both. A person is chosen at random. Find the probability that the person recycles either paper or plastic but not both.

(Total for Question 3 is 3 marks)

4 Let U be a group of 100 students. Let A be the set who study French and B be the set who study German. It is known that n(A) = 45, n(B) = 30 and n(A union B) = 60. Work out n(A intersect B).

(Total for Question 4 is 4 marks)

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5 Show that for any two finite sets A and B, n(A union B) = n(A) + n(B) - n(A intersect B) by considering the regions in a two-set Venn diagram. You do not need to give a diagram but give a concise explanation with arithmetic using variables.

(Total for Question 5 is 3 marks)

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6 State and prove De Morgan's law for two sets A and B: (A union B)^c = A^c intersect B^c. Give a short set membership argument showing both inclusions.

(Total for Question 6 is 4 marks)

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7 In a college, 200 students study at least one of three subjects M, N, O. The numbers who study M, N and O are 110, 95 and 80 respectively. Exactly 50 study both M and N, exactly 35 study both N and O, and exactly 30 study both M and O. Let x be the number who study all three. Find x, given that every student studies at least one subject and the counts above are accurate.

(Total for Question 7 is 5 marks)

8 Show that for three finite sets A, B and C the inclusion-exclusion formula n(A union B union C) = n(A) + n(B) + n(C) - n(A intersect B) - n(B intersect C) - n(A intersect C) + n(A intersect B intersect C) holds, by labelling the seven non-overlapping regions of a three-set Venn diagram with variables and summing.

(Total for Question 8 is 3 marks)

9 A survey of 200 customers asked which of three subscription services S, T and U they use. The numbers are: n(S)=120, n(T)=100, n(U)=80; n(S intersect T)=60, n(T intersect U)=50, n(S intersect U)=55. Exactly 20 customers subscribe to none. Let t be the number subscribing to all three.

(a) Find t.

(b) Find how many customers subscribe to exactly two services.

(c) Find the probability that a randomly chosen customer subscribes to at least two services.

(d) Find the probability that a randomly chosen customer subscribes to exactly one service.

(a) Find t, the number subscribing to all three. **(3)**

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(b) Find how many customers subscribe to exactly two services. **(3)**

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(c) Find the probability that a randomly chosen customer subscribes to at least two services. **(2)**

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(d) Find the probability that a randomly chosen customer subscribes to exactly one service. **(4)**

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(Total for Question 9 is 12 marks)

Mark scheme · 5.23H Venn Diagrams: Higher Tier Practice

Question 1

- **M1** Correctly fill Venn regions: only chess = 18 - 6 = 12; only football = 14 - 6 = 8; both = 6
- **A1** 4 students play neither (30 - (12+6+8) = 4) cao
- **Answer: 4 (neither).**

Question 2

- **M1** Place triple intersection = 10; compute pair-only regions: A and B only = 30 - 10 = 20, B and C only = 20 - 10 = 10, A and C only = 15 - 10 = 5
- **M1** Compute single-only counts, e.g. apples only = 50 - (20+5+10) = 15; bananas only = 70 - (20+10+10) = 30; cherries only = 40 - (10+10+5) = 15
- **M1** Add all regions to find number buying at least one: 15+30+15+20+10+5+10 = 105
- **A1** None = 120 - 105 = 15 cao
- **Answer: 15 shoppers buy none.**

Question 3

- **M1** Compute exclusive counts: paper only = 260 - 120 = 140; plastic only = 180 - 120 = 60
- **A1** Number recycling exactly one = 140 + 60 = 200
- **A1** Probability = 200/400 = 1/2 cao
- **Answer: 1/2**

Question 4

- **M1** Use formula n(A union B) = n(A) + n(B) - n(A intersect B)
- **M1** Substitute values: 60 = 45 + 30 - n(A intersect B)
- **M1** Rearrange to isolate n(A intersect B): n(A intersect B) = 45 + 30 - 60
- **A1** n(A intersect B) = 15 cao
- **Answer: 15**

Question 5

- **M1** Let x = n(A only), y = n(B only), z = n(A intersect B); then n(A) = x+z and n(B)=y+z
- **M1** Add: n(A)+n(B) = x+z+y+z = x+y+2z
- **A1** Since n(A union B) = x+y+z, conclude n(A union B) = n(A)+n(B)-z = n(A)+n(B)-n(A intersect B) cao
- **Answer: Proof: let A only = x, B only = y, A cap B = z. Then n(A)=x+z, n(B)=y+z so n(A)+n(B)=x+y+2z. The union has x+y+z, so n(A union B)=n(A)+n(B)-z = n(A)+n(B)-n(A intersect B).**

Question 6

- **B1** State law: (A union B)^c = A^c intersect B^c
- **M1** Show element-wise: if x in (A union B)^c then x not in A and x not in B, so x in A^c and x in B^c
- **M1** Converse: if x in A^c intersect B^c then x not in A and not in B, so x not in A union B, hence x in (A union B)^c
- **A1** Conclude equality of sets cao
- **Answer: Statement and proof: see mark scheme. Conclude (A union B)^c = A^c intersect B^c.**

Question 7

- **M1** Use inclusion-exclusion for three sets: n(M union N union O) = sum singles - sum pairs + n(all three)
- **M1** Substitute known values: 200 = 110 + 95 + 80 - (50 + 35 + 30) + x
- **M1** Compute sums: 110+95+80 = 285 and 50+35+30 = 115, so 200 = 285 - 115 + x
- **M1** Simplify to isolate x: 200 = 170 + x
- **A1** x = 30 cao
- **Answer: 30**

Question 8

- **M1** Let the seven regions be a, b, c, d, e, f, g where d is the triple intersection; express n(A), n(B), n(C) and pair intersections in terms of these
- **M1** Add n(A)+n(B)+n(C) and subtract the three pair intersections to obtain (sum of seven regions) + d
- **A1** Conclude inclusion-exclusion gives union = sum singles - sum pairs + triple intersection cao
- **Answer: Label regions a,b,c (single-only), e,f,g (pair-only), d (triple). Then n(A)+n(B)+n(C) - (pair sums) + d reduces to a+b+c+d+e+f+g, which is n(A union B union C) as required.**

Question 9

- (a) **M1** Use inclusion-exclusion: union = sum singles - sum pairs + t
- (a) **M1** Substitute values: n(union) = 200 - 20 = 180 = 120+100+80 - (60+50+55) + t
- (a) **A1** Compute t: 180 = 300 - 165 + t so t = 45 cao
- **(a) Answer: 45**
- (b) **M1** Compute pair-only counts: S and T only = 60 - 45 = 15, T and U only = 50 - 45 = 5, S and U only = 55 - 45 = 10
- (b) **M1** Add pair-only values to get exactly two: 15 + 5 + 10 = 30
- (b) **A1** Exactly two = 30 cao
- **(b) Answer: 30**
- (c) **M1** At least two = exactly two + triple = 30 + 45 = 75
- (c) **A1** Probability = 75/200 = 3/8 cao
- **(c) Answer: 3/8**
- (d) **M1** Compute single-only counts: S only = 120 - (15+10+45) = 50
- (d) **M1** T only = 100 - (15+5+45) = 35; U only = 80 - (10+5+45) = 20
- (d) **M1** Add single-only counts: 50 + 35 + 20 = 105
- (d) **A1** Probability = 105/200 = 21/40 cao
- **(d) Answer: 21/40**