HIGHER

5.9H Drawing Quadratic Graphs: Higher Tier Practice

Total Marks

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 65 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The graph of $y = x^2 - 4x + 3$ is a parabola. On graph paper, sketch this parabola. On your sketch, clearly mark the vertex and the x- and y-intercepts.

(Total for Question 1 is 3 marks)

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2 The parabola $y = -2(x - 1)^2 + 5$ is drawn. State the coordinates of the vertex and write down the equation of the axis of symmetry. Then sketch the graph, labelling the vertex and axis.

(Total for Question 2 is 3 marks)

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3 Find the equation of the parabola with vertex at (2, -3) that passes through the point (0, 1). Give your answer in the form $y = a(x - h)^2 + k$ and then expand it to $y = ax^2 + bx + c$.

(Total for Question 3 is 4 marks)

4 Solve algebraically the points of intersection between the parabola $y = x^2 - x - 6$ and the line $y = 2x + 3$. Give exact answers.

(Total for Question 4 is 4 marks)

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5 Show that for any quadratic $y = ax^2 + bx + c$ (a not equal to 0), the axis of symmetry of its graph is $x = -b/(2a)$, and that the y-intercept is c. You must show the algebraic steps (show that question).

(Total for Question 5 is 4 marks)

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6 For which value(s) of k does the quadratic $y = x^2 + 2kx + k + 1$ have exactly one x-intercept? Show your working and give exact values.

(Total for Question 6 is 3 marks)

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7 State the range of the quadratic $y = -(x - 3)^2 + 7$. Justify your answer in one sentence.

(Total for Question 7 is 2 marks)

8 Consider the two graphs $y = x^2 - 4$ and $y = 2 - x^2/2$.

(a) Algebraically find the points of intersection. (3 marks)

(b) State how many points of intersection there are and explain briefly why the number of intersections matches the algebraic result. (2 marks)

(a) Algebraically find the points of intersection. **(3)**

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(b) State how many points of intersection there are and explain briefly why. **(2)**

(Total for Question 8 is 5 marks)

Mark scheme · 5.9H Drawing Quadratic Graphs: Higher Tier Practice

Question 1

- **M1** complete sketch showing a parabola with correct shape and axis of symmetry x = 2 (vertex visible) ft
- **A1** vertex marked at (2, -1) cao
- **A1** x-intercepts at x = 1 and x = 3 and y-intercept at (0, 3) all marked cao
- **Answer: Vertex (2, -1); x-intercepts (1, 0) and (3, 0); y-intercept (0, 3).**

Question 2

- **M1** state vertex from vertex form or identify axis x = 1 ft
- **A1** vertex (1, 5) cao
- **A1** axis of symmetry x = 1 cao
- **Answer: Vertex (1, 5); axis of symmetry x = 1.**

Question 3

- **M1** use vertex form $y = a(x - 2)^2 - 3$ and substitute (0,1) to form an equation in a
- **A1** a = 1 cao
- **M1** write vertex form with found a: $y = 1(x - 2)^2 - 3$ (oe)
- **A1** expanded form $y = x^2 - 4x + 1$ cao
- **Answer: $y = (x - 2)^2 - 3$; expanded $y = x^2 - 4x + 1$.**

Question 4

- **M1** equate and rearrange to form quadratic $x^2 - 3x - 9 = 0$
- **M1** use quadratic formula correctly: $x = (3 \pm \sqrt{45})/2$
- **A1** simplify $\sqrt{45}$ to 3sqrt(5) or leave as $\sqrt{45}$ and give $x = (3 \pm 3\text{sqrt}(5))/2$ cao
- **A1** give corresponding y values: $y = 2x + 3$, so $y = 2((3 \pm 3\text{sqrt}(5))/2) + 3 = 3 \pm 3\text{sqrt}(5) + 3 = 6 \pm 3\text{sqrt}(5)$ cao
- **Answer: Intersection points: ((3 + 3sqrt(5))/2, 6 + 3sqrt(5)) and ((3 - 3sqrt(5))/2, 6 - 3sqrt(5)).**

Question 5

- **dM1** complete the square: $ax^2 + bx + c = a(x^2 + (b/a)x) + c = a[(x + b/(2a))^2 - (b/(2a))^2] + c$ (dep)
- **M1** rearrange to vertex form $y = a(x + b/(2a))^2 + (c - b^2/(4a))$
- **A1** axis of symmetry x = -b/(2a) cao
- **A1** y-intercept is c since at x = 0, y = c cao
- **Answer: Completing the square gives $y = a(x + b/(2a))^2 + (c - b^2/(4a))$, so axis x = -b/(2a). Also y(0)=c, so y-intercept is (0, c).**

Question 6

- **M1** use discriminant $b^2 - 4ac = 0$ condition for one x-intercept: $(2k)^2 - 4(1)(k + 1) = 0$
- **M1** simplify to $4(k^2 - k - 1) = 0$ and solve quadratic $k^2 - k - 1 = 0$
- **A1** $k = (1 \pm \sqrt{5})/2$ cao
- **Answer: $k = (1 + \sqrt{5})/2$ or $k = (1 - \sqrt{5})/2$.**

Question 7

- **B1** state range $y \le 7$ or (-infinity, 7] oe
- **B1** reason: vertex at (3, 7) is a maximum since coefficient is negative oe
- **Answer: Range: $y \le 7$ because the vertex (3, 7) is a maximum (leading coefficient negative).**

Question 8

- (a) **M1** equate $x^2 - 4 = 2 - x^2/2$ and rearrange correctly to form $3x^2 = 12$ or equivalent
- (a) **M1** solve $x^2 = 4$ to get $x = \pm 2$
- (a) **A1** give intersection points (2, 0) and (-2, 0) cao
- **(a) Answer: (2, 0) and (-2, 0)**
- (b) **B1** state there are 2 points of intersection cao
- (b) **B1** explain because the rearranged equation $3x^2 - 12 = 0$ gives two distinct real solutions $x = \pm 2$ oe
- **(b) Answer: There are 2 intersections because the algebraic equation gives two distinct real solutions $x = \pm 2$, leading to two distinct points.**