HIGHER

6.10 Circle Theorems: 2-hour Pack

EDEXCEL 1MA1 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A, B and C are points on the circumference of a circle with centre O. C is on the major arc AB. Angle ACB = 30 degrees. Work out the size of angle AOB. Give a reason for your answer.

(Total for Question 1 is 2 marks)

---

2 PT is a tangent to a circle with centre O at the point T. P is outside the circle. Angle TPO = 35 degrees. Work out the size of angle POT. Give reasons for your working.

(Total for Question 2 is 3 marks)

---

3 T and A are points on a circle. The tangent at T makes an angle of 36 degrees with the chord TA. B is a point on the major arc TA. Work out the size of angle TBA. Give a reason for your answer.

(Total for Question 3 is 2 marks)

---

4 A, B, C and D are points on a circle, in that order, so that ABCD is a cyclic quadrilateral. Angle ABC = 74 degrees. Work out the size of angle ADC. Give a reason for your answer.

(Total for Question 4 is 2 marks)

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5 A, B and C are points on the circumference of a circle with centre O. C is on the major arc AB. Angle AOB = 80 degrees. Work out the size of angle ACB. Give a reason for your answer.

(Total for Question 5 is 2 marks)

---

6 PT is a tangent to a circle with centre O at the point T. P is outside the circle. Angle TPO = 60 degrees. Work out the size of angle POT. Give reasons for your working.

(Total for Question 6 is 3 marks)

---

7 PA and PB are tangents from the point P to a circle with centre O, touching the circle at A and B. Angle APB = 108 degrees. Work out the size of angle AOB.

(Total for Question 7 is 3 marks)

---

8 PT is a tangent to a circle with centre O at the point T. P is outside the circle. Angle TPO = 41 degrees. Work out the size of angle POT. Give reasons for your working.

(Total for Question 8 is 3 marks)

---

9 A, B, C and D are points on a circle, in that order, so that ABCD is a cyclic quadrilateral. Angle ABC = 82 degrees. Work out the size of angle ADC. Give a reason for your answer.

(Total for Question 9 is 2 marks)

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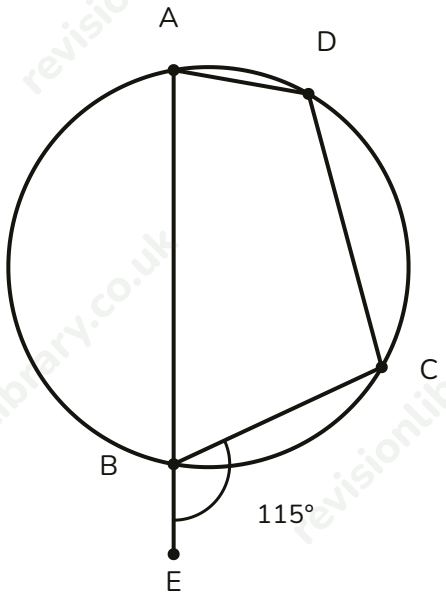
10 T and A are points on a circle with centre O. The tangent at T makes an angle of 44 degrees with the chord TA. Work out the size of angle TOA. Give reasons for your working.

(Total for Question 10 is 3 marks)

---

11 ABCD is a cyclic quadrilateral. Side AB is extended to a point E, so that angle CBE is the exterior angle of the quadrilateral at B. Angle CBE = 115 degrees.

Diagram NOT accurately drawn



(a) Work out the size of angle ADC. Give a reason for your answer. **(2)**

.....

(b) Work out the size of angle ABC. **(2)**

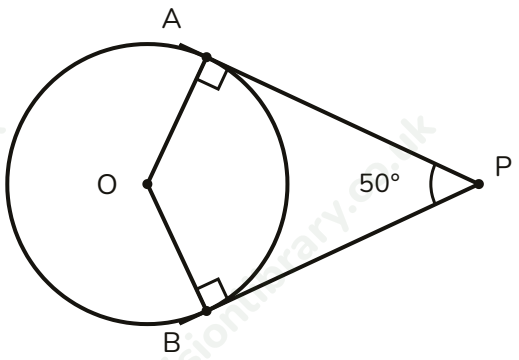
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(Total for Question 11 is 4 marks)

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12 PA and PB are tangents to a circle with centre O, touching the circle at A and B respectively.
Angle APB = 50 degrees.

Diagram NOT accurately drawn



(a) Work out the size of angle AOB. **(3)**

.....

(b) Work out the size of angle OAB. **(2)**

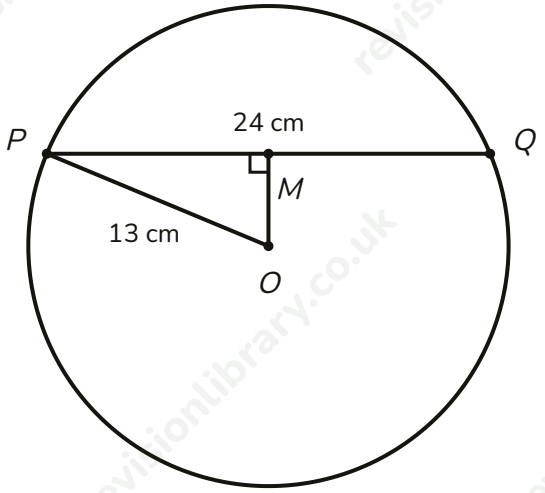
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(Total for Question 12 is 5 marks)

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13 A chord PQ of a circle with centre O has length 24 cm. The radius of the circle is 13 cm. M is the midpoint of PQ, and OM is drawn perpendicular to PQ.

Diagram NOT accurately drawn



(a) Explain why PM = 12 cm. **(1)**

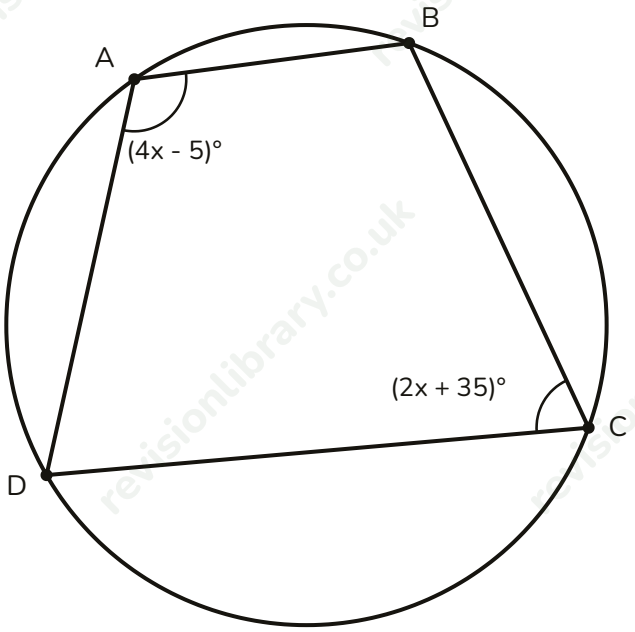
(b) Work out the length OM. **(2)**

(Total for Question 13 is 3 marks)

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14 ABCD is a cyclic quadrilateral. Angle A = (4x - 5) degrees and angle C = (2x + 35) degrees.

Diagram NOT accurately drawn



(a) Explain why angle A + angle C = 180 degrees. **(1)**

(b) Work out the value of x. **(2)**

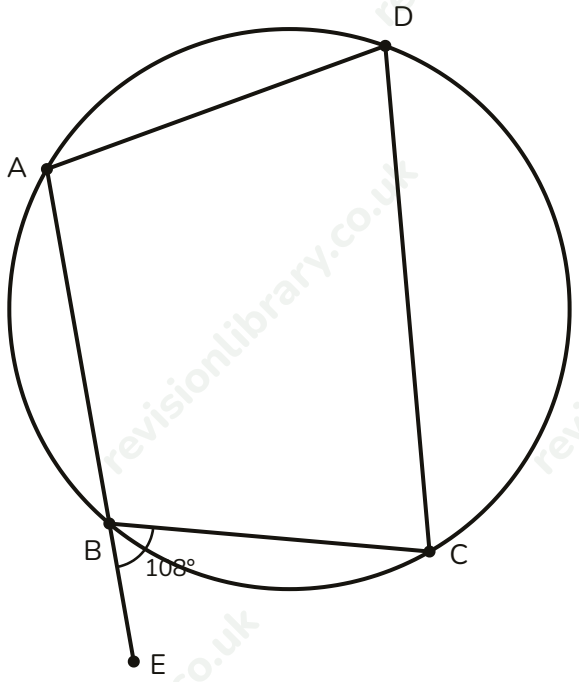
(c) Work out the size of angle A and the size of angle C. **(2)**

(Total for Question 14 is 5 marks)

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15 ABCD is a cyclic quadrilateral. Side AB is extended to a point E, so that angle CBE is the exterior angle of the quadrilateral at B. Angle CBE = 108 degrees.

Diagram NOT accurately drawn



(a) Work out the size of angle ADC. Give a reason for your answer. **(2)**

.....

(b) Work out the size of angle ABC. **(2)**

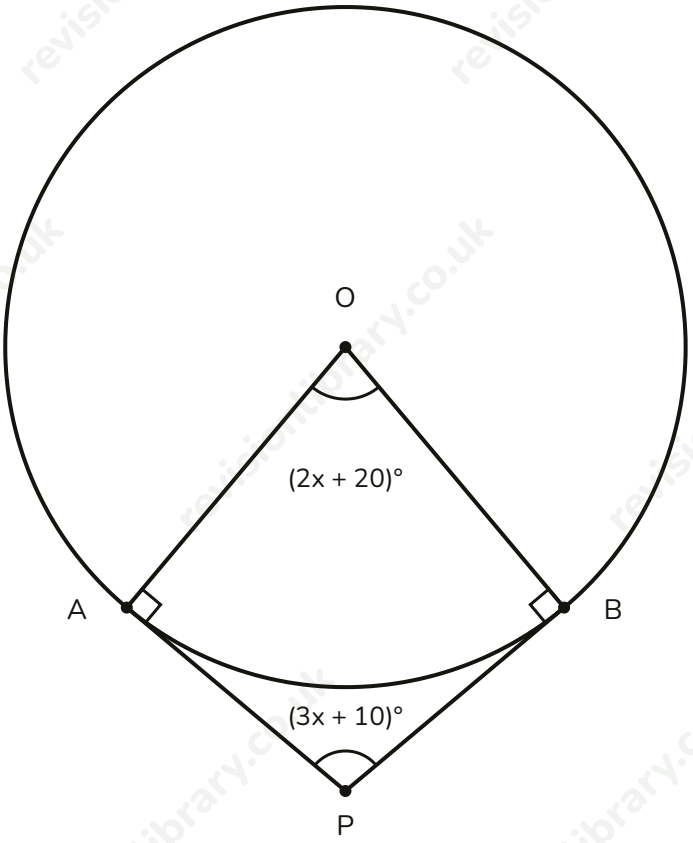
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(Total for Question 15 is 4 marks)

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16 PA and PB are tangents to a circle with centre O, touching the circle at A and B. Angle APB = (3x + 10) degrees and angle AOB = (2x + 20) degrees.

Diagram NOT accurately drawn



(a) Show that angle APB + angle AOB = 180 degrees. **(2)**

(b) Work out the value of x. **(2)**

.....

(c) Work out the size of angle APB. **(1)**

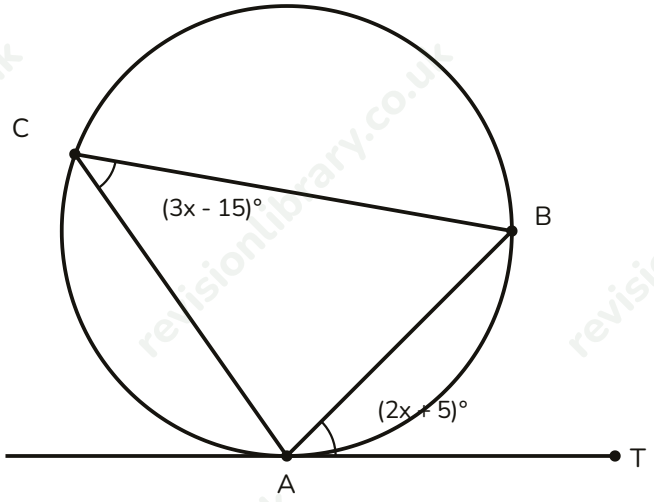
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(Total for Question 16 is 5 marks)

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17 TA is a tangent to a circle at the point A. AB and AC are chords of the circle. The angle between the tangent and the chord AB is (2x + 5) degrees. The angle in the alternate segment, angle ACB, is (3x - 15) degrees.

Diagram NOT accurately drawn



(a) Use the alternate segment theorem to form an equation in x, and solve it to find the value of x. **(2)**

.....

(b) Work out the size of angle ACB. **(1)**

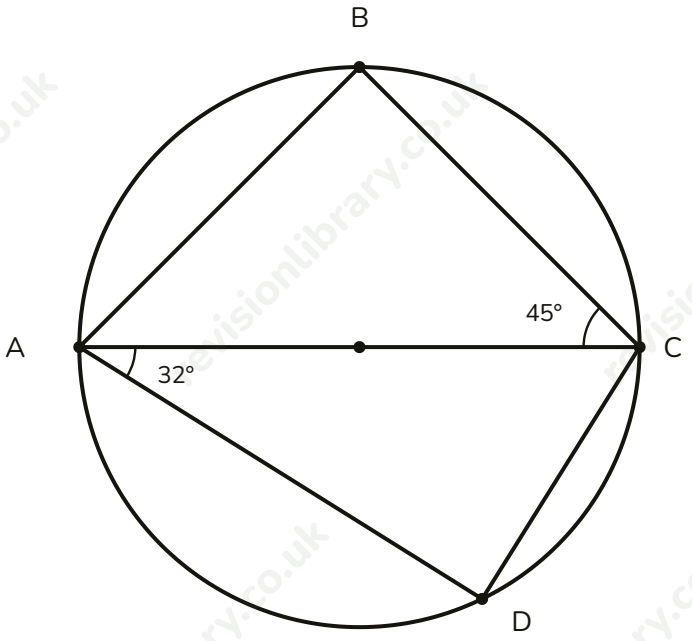
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(Total for Question 17 is 3 marks)

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18 A, B, C and D are points on a circle, with AC a diameter of the circle. B and D lie on opposite sides of AC. Angle ACB = 45 degrees and angle CAD = 32 degrees.
Work out the size of angle BAD and the size of angle BCD, giving reasons at each stage of your working.

Diagram NOT accurately drawn

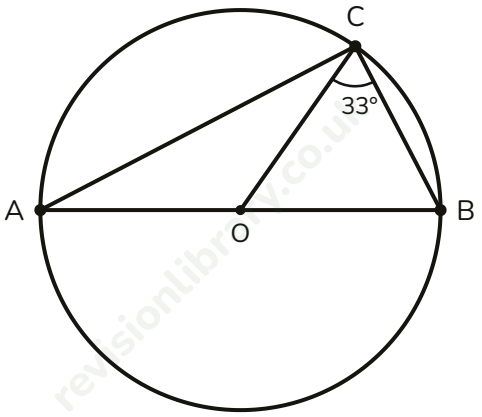


(Total for Question 18 is 6 marks)

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19 AB is a diameter of a circle with centre O. C is a point on the circumference, and OC is drawn.
Angle OCB = 33 degrees.
Work out the size of angle BAC, giving reasons at each stage of your working.

Diagram NOT accurately drawn

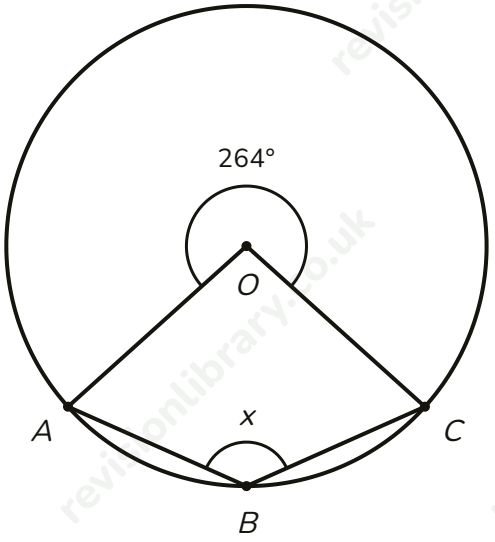


(Total for Question 19 is 4 marks)

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20 A, B and C are points on the circumference of a circle with centre O. B lies on the minor arc AC. The reflex angle AOC = 264 degrees.

Diagram NOT accurately drawn



(a) Work out the size of the non-reflex angle AOC. **(1)**

.....

(b) Work out the size of angle ABC. **(3)**

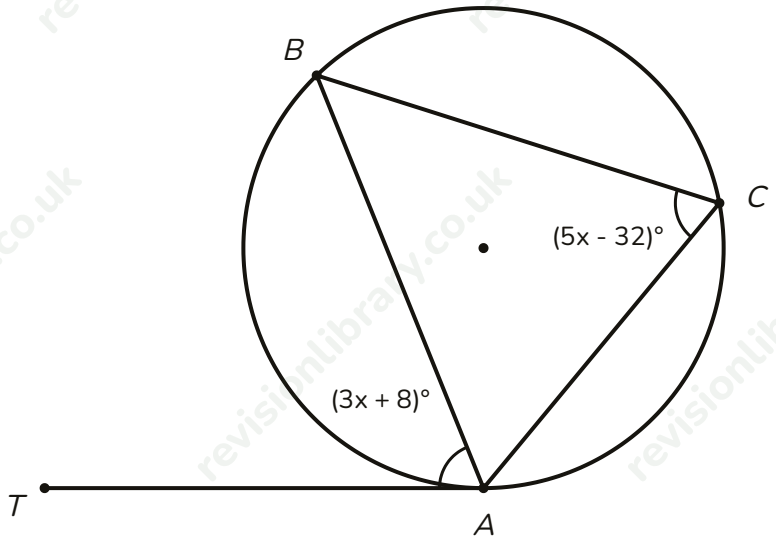
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(Total for Question 20 is 4 marks)

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21 TA is a tangent to a circle at the point A. AB and AC are chords of the circle. The angle between the tangent and the chord AB is (3x + 8) degrees. The angle in the alternate segment, angle ACB, is (5x - 32) degrees.

Diagram NOT accurately drawn



(a) Use the alternate segment theorem to form an equation in x, and solve it to find the value of x. **(2)**

.....

(b) Work out the size of angle ACB. **(1)**

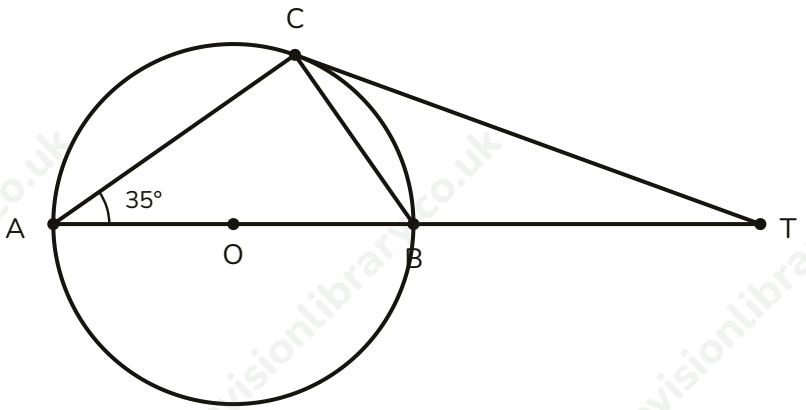
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(Total for Question 21 is 3 marks)

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22 AB is a diameter of a circle, centre O. C is a point on the circle. The tangent to the circle at C meets the line AB extended beyond B at the point T. Angle CAB = 35 degrees.
Work out the size of angle CTB, giving reasons at each stage of your working.

Diagram NOT accurately drawn

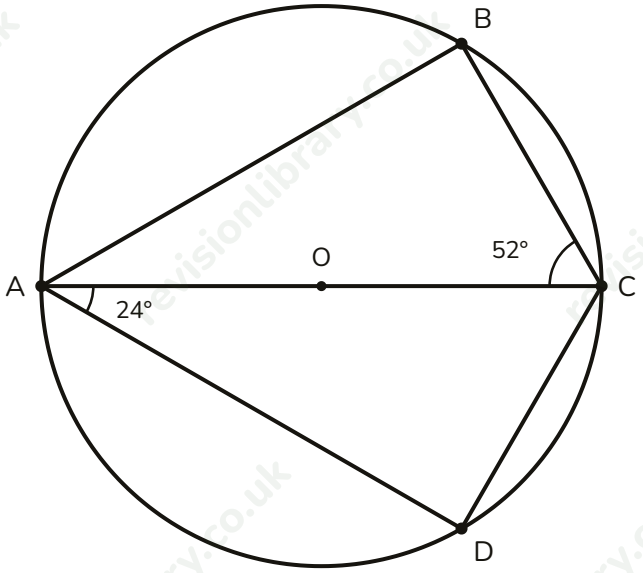


(Total for Question 22 is 6 marks)

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23 A, B, C and D are points on a circle, with AC a diameter of the circle. B and D lie on opposite sides of AC. Angle ACB = 52 degrees and angle CAD = 24 degrees.
Work out the size of angle BAD and the size of angle BCD, giving reasons at each stage of your working.

Diagram NOT accurately drawn

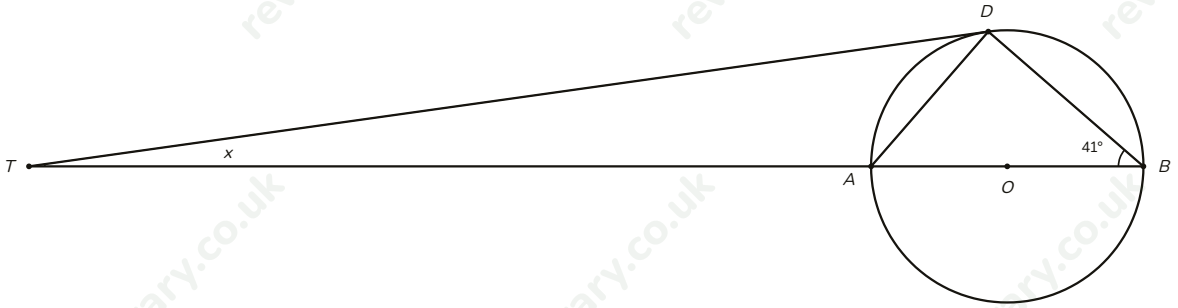


(Total for Question 23 is 6 marks)

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24 AB is a diameter of a circle, centre O. D is a point on the circle. The tangent to the circle at D meets the line BA extended beyond A at the point T. Angle DBA = 41 degrees.
Work out the size of angle DTA, giving reasons at each stage of your working.

Diagram NOT accurately drawn



(Total for Question 24 is 6 marks)

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Mark scheme · 6.10 Circle Theorems: 2-hour Pack

Question 1

60 degrees **B1**

reason: the angle at the centre is twice the angle at the circumference **B1**

Answer: 60 degrees

Question 2

angle OTP = 90 degrees (tangent meets radius at 90 degrees) **B1**

180 - 90 - 35 **M1**

55 degrees cao **A1**

Answer: 55 degrees

Question 3

36 degrees **B1**

reason: alternate segment theorem **B1**

Answer: 36 degrees

Question 4

106 degrees **B1**

reason: opposite angles of a cyclic quadrilateral add up to 180 degrees **B1**

Answer: 106 degrees

Question 5

40 degrees **B1**

reason: the angle at the centre is twice the angle at the circumference **B1**

Answer: 40 degrees

Question 6

angle OTP = 90 degrees (tangent meets radius at 90 degrees) **B1**

180 - 90 - 60 **M1**

30 degrees cao **A1**

Answer: 30 degrees

Question 7

angles OAP and OBP are each 90 degrees (tangent meets radius at 90 degrees) **B1**

360 - 90 - 90 - 108 **M1**

72 degrees cao **A1**

Answer: 72 degrees

Question 8

angle OTP = 90 degrees (tangent meets radius at 90 degrees) **B1**

180 - 90 - 41 **M1**

49 degrees cao **A1**

Answer: 49 degrees

Question 9

98 degrees **B1**

reason: opposite angles of a cyclic quadrilateral add up to 180 degrees **B1**

Answer: 98 degrees

Question 10

angle OTA = 90 - 44 (tangent and radius meet at 90 degrees) **M1**

triangle OTA is isosceles so angle OAT is equal **M1**

88 degrees cao **A1**

Answer: 88 degrees

Question 11

(a) use the exterior angle of a cyclic quadrilateral equals the interior opposite angle **M1**

(a) 115 degrees cao, with reason stated **A1**

(a) Answer: 115 degrees

(b) 180 - 115 oe, using angles on a straight line ABE or opposite angles in a cyclic quadrilateral **M1**

(b) 65 degrees cao **A1**

(b) Answer: 65 degrees

Question 12

(a) angle OAP = angle OBP = 90 degrees (tangent perpendicular to radius) **M1**

(a) 360 - 90 - 90 - 50 oe (angle sum of quadrilateral OAPB) **M1**

(a) 130 degrees cao **A1**

(a) Answer: 130 degrees

(b) (180 - their AOB) / 2 oe, using triangle OAB isosceles since OA = OB (radii) **M1**

(b) 25 degrees cao **A1**

(b) Answer: 25 degrees

Question 13

(a) the perpendicular from the centre to a chord bisects the chord, so PM = 24 / 2 = 12 cm oe **B1**

(a) Answer: The perpendicular from the centre to a chord bisects the chord, so PM = 24 / 2 = 12 cm.

(b) $\sqrt{13^2 - 12^2}$ oe **M1**

(b) 5 cm cao **A1**

(b) Answer: 5 cm

Question 14

(a) opposite angles in a cyclic quadrilateral sum to 180 degrees oe **B1**

(a) Answer: Opposite angles in a cyclic quadrilateral sum to 180 degrees, and A and C are opposite angles.

(b) 4x - 5 + 2x + 35 = 180 oe **M1**

(b) x = 25 cao **A1**

(b) Answer: x = 25

(c) substitute their x into 4x - 5 and 2x + 35 (ft) **M1**

(c) angle A = 95 degrees and angle C = 85 degrees, both cao **A1**

(c) Answer: Angle A = 95 degrees, angle C = 85 degrees

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Question 15

(a) use the exterior angle of a cyclic quadrilateral equals the interior opposite angle **M1**

(a) 108 degrees cao, with reason stated **A1**

(a) Answer: 108 degrees

(b) 180 - 108 oe, using angles on a straight line ABE or opposite angles in a cyclic quadrilateral **M1**

(b) 72 degrees cao **A1**

(b) Answer: 72 degrees

Question 16

(a) angle OAP = angle OBP = 90 degrees (tangent perpendicular to radius) **M1**

(a) angle sum of quadrilateral OAPB = 360, so angle APB + angle AOB = 360 - 90 - 90 = 180 degrees cso **A1**

(a) Answer: Angle APB + angle AOB = 180 degrees (shown)

(b) (3x + 10) + (2x + 20) = 180 oe **M1**

(b) x = 30 cao **A1**

(b) Answer: x = 30

(c) 100 degrees cao (ft their x) **B1**

(c) Answer: 100 degrees

Question 17

(a) 2x + 5 = 3x - 15 oe (equating tangent-chord angle to angle in alternate segment) **M1**

(a) x = 20 cao **A1**

(a) Answer: x = 20

(b) 45 degrees cao (ft their x) **B1**

(b) Answer: 45 degrees

Question 18

angle ABC = 90 degrees, since AC is a diameter (angle in a semicircle) **B1**

angle BAC = 180 - 90 - 45 = 45 degrees (angle sum of triangle ABC) **M1**

angle ADC = 90 degrees, since AC is a diameter (angle in a semicircle) **B1**

angle ACD = 180 - 90 - 32 = 58 degrees (angle sum of triangle ACD) **M1**

angle BAD = angle BAC + angle CAD = 45 + 32 = 77 degrees cao **A1**

angle BCD = angle BCA + angle ACD = 45 + 58 = 103 degrees cao (ft) **A1**

Answer: Angle BAD = 77 degrees, angle BCD = 103 degrees

Question 19

angle ACB = 90 degrees, since AB is a diameter (angle in a semicircle) **B1**

angle OCA = 90 - 33 = 57 degrees **M1**

OA = OC (radii), so triangle OAC is isosceles, angle OAC = angle OCA **M1**

angle BAC = 57 degrees cao **A1**

Answer: 57 degrees

Question 20

(a) 96 degrees cao **B1**

(a) Answer: 96 degrees

(b) identify that the angle at the centre theorem applies to the reflex angle AOC **M1**

(b) 132 degrees cao **A1**

(b) reason: the angle at the centre is twice the angle at the circumference, subtended by the same (major) arc AC oe **B1**

(b) Answer: 132 degrees

Question 21

(a) 3x + 8 = 5x - 32 oe (equating tangent-chord angle to angle in alternate segment) **M1**

(a) x = 20 cao **A1**

(a) Answer: x = 20

(b) 68 degrees cao (ft their x) **B1**

(b) Answer: 68 degrees

Question 22

angle ACB = 90 degrees, since AB is a diameter (angle in a semicircle) **B1**

angle ABC = 180 - 90 - 35 = 55 degrees (angle sum of triangle ABC) **M1**

angle CBT = 180 - 55 = 125 degrees (angles on the straight line ABT) **M1**

angle BCT = 35 degrees (alternate segment theorem: tangent-chord angle at C between CT and chord CB equals angle CAB in the alternate segment) **B1**

angle CTB = 180 - 125 - 35 (angle sum of triangle BCT) **M1**

20 degrees cao **A1**

Answer: 20 degrees

Question 23

angle ABC = 90 degrees, since AC is a diameter (angle in a semicircle) **B1**

angle BAC = 180 - 90 - 52 = 38 degrees (angle sum of triangle ABC) **M1**

angle ADC = 90 degrees, since AC is a diameter (angle in a semicircle) **B1**

angle ACD = 180 - 90 - 24 = 66 degrees (angle sum of triangle ACD) **M1**

angle BAD = angle BAC + angle CAD = 38 + 24 = 62 degrees cao **A1**

angle BCD = angle BCA + angle ACD = 52 + 66 = 118 degrees cao (ft) **A1**

Answer: Angle BAD = 62 degrees, angle BCD = 118 degrees

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Question 24

angle ADB = 90 degrees, since AB is a diameter (angle in a semicircle)	**B1**
angle DAB = 180 - 90 - 41 = 49 degrees (angle sum of triangle ABD)	**M1**
angle DAT = 180 - 49 = 131 degrees (angles on the straight line TAB)	**M1**
angle ADT = 41 degrees (alternate segment theorem: tangent-chord angle at D between DT and chord DA equals angle DBA in the alternate segment)	**B1**
angle DTA = 180 - 131 - 41 (angle sum of triangle ADT)	**M1**
8 degrees cao	**A1**

Answer: 8 degrees

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