HIGHER

7.10 Iteration: Test Pack

EDEXCEL 1MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Show that the equation $f(x) = x^3 - 2x - 1$ has a root between $x = 1$ and $x = 2$.

(Total for Question 1 is 1 mark)

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2 Use the fixed-point iteration $x_{n+1} = (x_n + 3/x_n)/2$ to approximate $\sqrt{3}$. Start with $x_0 = 2$. Find x_1 and x_2, each to 3 decimal places.

(Total for Question 2 is 3 marks)

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3 Consider the equation $x^3 + 4x - 10 = 0$ and the iteration $x_{n+1} = (10 - x_n^3)/4$. Starting with $x_0 = 1.5$, calculate x_1 and x_2. From these two iterations, state whether the sequence appears to be converging.

(Total for Question 3 is 3 marks)

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4 Use the iteration $x_{n+1} = \cos(x_n)$ with $x_0 = 1$. Find x_2 and x_3 to 3 decimal places. From the trend of values, state the value the sequence appears to be approaching (to 3 d.p.).

(Total for Question 4 is 4 marks)

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5 The equation $x^2 + x - 6 = 0$ has a positive root. Rearrange it to the fixed-point form $x = 6/(x + 1)$. Starting with $x_0 = 1.5$, perform two iterations using $x_{n+1} = 6/(x_n + 1)$. Give x_2 to 3 decimal places and say whether the values appear to be approaching the positive root.

(Total for Question 5 is 3 marks)

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6 $f(x) = x^3 + 5x - 10$. Show that the equation $f(x) = 0$ has a root between $x = 1$ and $x = 2$.

(Total for Question 6 is 3 marks)

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7 $f(x) = x^3 - x - 40$. The equation $f(x) = 0$ has a root between $x = 3$ and $x = 4$. By evaluating $f(3.5)$, decide whether the root is between 3 and 3.5 or between 3.5 and 4.

(Total for Question 7 is 3 marks)

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8 A sequence is defined by $x_0 = 4$ and $x_{n+1} = \sqrt{2x_n + 9}$. Work out x_1 and x_2. Give your answers correct to 3 decimal places.

(Total for Question 8 is 3 marks)

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9 Show that the equation $x^3 - 5x - 1 = 0$ can be rearranged into the form $x = (px + q)^{1/3}$, where p and q are integers. State the rearranged equation.

(Total for Question 9 is 2 marks)

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10 $f(x) = x^3 + 5x - 24$. The equation $f(x) = 0$ has a root between $x = 2$ and $x = 3$. By evaluating $f(2.5)$, decide whether the root is between 2 and 2.5 or between 2.5 and 3.

(Total for Question 10 is 3 marks)

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11 $f(x) = x^3 + 9x - 36$. The equation $f(x) = 0$ has a root between $x = 2$ and $x = 3$. By evaluating $f(2.5)$, decide whether the root is between 2 and 2.5 or between 2.5 and 3.

(Total for Question 11 is 3 marks)

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12 Use Newton-Raphson to approximate the cube root of 7 by finding a root of $f(x) = x^3 - 7$. Start with $x_0 = 2$. Perform two iterations and give x_2 to 4 decimal places.

(Total for Question 12 is 4 marks)

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13 Use the Newton-Raphson method to find a root of $f(x) = x^3 - x - 2$. Start with $x_0 = 1.5$. Perform two iterations and give your result x_2 to 4 decimal places.

(Total for Question 13 is 4 marks)

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14 Show that $f(x) = x^3 - 2x - 5$ has a root between $x = 2$ and $x = 3$. Then use the iteration $x_{n+1} = (5 + 2x_n)^{1/3}$ with $x_0 = 2.5$ to compute x_1 and x_2. Comment briefly on whether this rearrangement seems to produce convergence.

(Total for Question 14 is 4 marks)

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Mark scheme · 7.10 Iteration: Test Pack

Question 1

f(1) = -2 and f(2) = 3 so signs differ, hence a root in (1, 2) cao — **B1**

Answer: f(1) = 1 - 2 - 1 = -2, f(2) = 8 - 4 - 1 = 3. Since f(1) < 0 and f(2) > 0 there is a root between 1 and 2 (by intermediate value theorem).

Question 2

substitute $x_0 = 2$ into $x_{n+1} = (x_n + 3/x_n)/2$ and calculate x_1 — **M1**

use x_1 to calculate x_2 with the same formula — **M1**

$x_1 = 1.750$, $x_2 = 1.732$ cao — **A1**

Answer: $x_1 = (2 + 3/2)/2 = 1.75$ -> 1.750; $x_2 = (1.75 + 3/1.75)/2$ approx 1.732142857 -> 1.732

Question 3

calculate $x_1 = (10 - 1.5^3)/4 = 1.65625$ — **M1**

calculate $x_2 = (10 - x_1^3)/4 = 1.36415863$ (awrt) — **M1**

conclude that the sequence does not appear to be converging (it oscillates) oe — **B1**

Answer: $x_1 = (10 - 1.5^3)/4 = 6.625/4 = 1.65625$; $x_2 = (10 - 1.65625^3)/4$ approx 1.36415863. The values jump (1.5 -> 1.65625 -> 1.36416), so the sequence appears to oscillate and not settle (not converging).

Question 4

calculate $x_1 = \cos(1)$ and $x_2 = \cos(x_1)$ — **M1**

calculate $x_3 = \cos(x_2)$ — **M1**

give x_2 and x_3 to 3 dp correctly — **M1**

state the sequence appears to approach 0.739 (awrt) cao — **A1**

Answer: $x_1 = \cos(1)$ approx 0.5403023059; $x_2 = \cos(0.5403023059)$ approx 0.8575532158 -> 0.858; $x_3 = \cos(0.8575532158)$ approx 0.6542897905 -> 0.654. The sequence appears to approach approximately 0.739 (the fixed point of cos x).

Question 5

calculate $x_1 = 6/(1.5 + 1) = 6/2.5 = 2.4$ — **M1**

calculate $x_2 = 6/(2.4 + 1) = 6/3.4 = 1.764705882$ — **M1**

$x_2 = 1.765$ (awrt); values appear to be moving toward the positive root x = 2 (comment) oe — **A1**

Answer: $x_1 = 2.4$; x_2 approx 1.764706 -> 1.765. The iterations move around the true positive root 2 and appear to be approaching it (oscillating but converging).

Question 6

f(1) evaluated — **M1**

f(2) evaluated — **M1**

values with a correct reason (change of sign) and conclusion — **A1**

Answer: f(1) = -4, f(2) = 8, a change of sign

Question 7

substitutes x = 3.5 — **M1**

f(3.5) = -0.625 — **A1**

correct interval 3.5 to 4 with a reason — **B1**

Answer: f(3.5) = -0.625, root between 3.5 and 4

Question 8

substitutes $x_0 = 4$ — **M1**

$x_1 = 4.123$ — **A1**

$x_2 = 4.153$ — **A1**

Answer: $x_1 = 4.123$, $x_2 = 4.153$

Question 9

isolates a term in x correctly — **M1**

$x = (5x + 1)^{1/3}$ — **A1**

Answer: $x = (5x + 1)^{1/3}$

Question 10

substitutes x = 2.5 — **M1**

f(2.5) = 4.125 — **A1**

correct interval 2 to 2.5 with a reason — **B1**

Answer: f(2.5) = 4.125, root between 2 and 2.5

Question 11

substitutes x = 2.5 — **M1**

f(2.5) = 2.125 — **A1**

correct interval 2 to 2.5 with a reason — **B1**

Answer: f(2.5) = 2.125, root between 2 and 2.5

Question 12

state Newton formula $x_{n+1} = x_n - (x_n^3 - 7)/(3x_n^2)$ — **M1**

calculate $x_1 = 2 - 1/12 = 1.9166666667$ — **M1**

calculate x_2 using x_1: x_2 approx 1.9129385 — **M1**

$x_2 = 1.9129$ (to 4 dp) cao — **A1**

Answer: $x_1 = 2 - (8 - 7)/12 = 1.9166666667$. x_2 approx $1.9166666667 - (1.9166666667^3 - 7)/(3*1.9166666667^2)$ approx 1.9129385 -> 1.9129 (4 dp).

Question 13

state Newton formula $x_{n+1} = x_n - f(x_n)/f'(x_n)$ with $f' = 3x^2 - 1$ — **M1**

calculate $x_1 = 1.5 - (-0.125)/5.75 = 35/23 = 1.5217391304$ — **M1**

calculate x_2 using x_1 (use $f(x_1) = 26/12167$ and $f'(x_1) = 3146/529$ or equivalent) — **M1**

$x_2 = 1.5214$ (to 4 dp) cao — **A1**

Answer: $x_1 = 35/23$ approx 1.5217391304. $f(x_1) = 26/12167$, $f'(x_1)=3146/529$, so x_2 approx 1.52137953 -> 1.5214 (4 dp).

Question 14

evaluate f(2) = 8 - 4 - 5 = -1 and f(3) = 27 - 6 - 5 = 16 — **M1**

calculate $x_1 = (5 + 2*2.5)^{1/3} = (10)^{1/3} = 10^{1/3}$ approx 2.154435 — **M1**

calculate $x_2 = (5 + 2*x_1)^{1/3}$ approx $(5 + 4.30887)^{1/3} = 9.30887^{1/3}$ approx 2.104 — **M1**

conclude whether iteration appears to converge (values move toward ~2.094...) cao — **A1**

Answer: f(2) = -1, f(3) = 16 so a root lies between 2 and 3. $x_1 = 10^{1/3}$ approx 2.154435; x_2 approx $9.30887^{1/3}$ approx 2.104. The values move toward about 2.094; the rearrangement appears to produce convergence.

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