7.10D Iteration: Fluency and Exam Drill

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: ______________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 Show that the equation $f(x) = x^3 - 2x - 1$ has a root between $x = 1$ and $x = 2$.

(Total for Question 1 is 1 mark)

2 Use the fixed-point iteration x_{n+1} = $(x_n + 3/x_n)/2$ to approximate $\sqrt{3}$. Start with $x_0 = 2$. Find x_1 and x_2, each to 3 decimal places.

(Total for Question 2 is 3 marks)

3 Use the iteration x_{n+1} = $(x_n + 2/x_n)/2$ to approximate $\sqrt{2}$. Start with $x_0 = 1.5$. Find x_2 to 3 decimal places.

(Total for Question 3 is 3 marks)

4 Show that the polynomial $f(x) = x^4 - 3x + 1$ has a root between $x = 0$ and $x = 1$.

(Total for Question 4 is 1 mark)

5 Consider the equation $x^3 + 4x - 10 = 0$ and the iteration x_{n+1} = $(10 - x_n^3)/4$. Starting with $x_0 = 1.5$, calculate x_1 and x_2. From these two iterations, state whether the sequence appears to be converging.

(Total for Question 5 is 3 marks)

6 Use the iteration x_{n+1} = $\cos(x_n)$ with $x_0 = 1$. Find x_2 and x_3 to 3 decimal places. From the trend of values, state the value the sequence appears to be approaching (to 3 d.p.).

(Total for Question 6 is 4 marks)

7 Use the Newton-Raphson method to find a root of $f(x) = x^3 - x - 2$. Start with $x_0 = 1.5$. Perform two iterations and give your result x_2 to 4 decimal places.

(Total for Question 7 is 4 marks)

8 The equation $x^2 + x - 6 = 0$ has a positive root. Rearrange it to the fixed-point form $x = 6/(x + 1)$. Starting with $x_0 = 1.5$, perform two iterations using x_{n+1} = $6/(x_n + 1)$. Give x_2 to 3 decimal places and say whether the values appear to be approaching the positive root.

(Total for Question 8 is 3 marks)

9 Show that $f(x) = x^3 - 2x - 5$ has a root between $x = 2$ and $x = 3$. Then use the iteration x_{n+1} $= (5 + 2x_n)^{1/3}$ with $x_0 = 2.5$ to compute x_1 and x_2. Comment briefly on whether this rearrangement seems to produce convergence.

(Total for Question 9 is 4 marks)

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10 Use Newton-Raphson to approximate the cube root of 7 by finding a root of $f(x) = x^3 - 7$. Start with $x_0 = 2$. Perform two iterations and give x_2 to 4 decimal places.

(Total for Question 10 is 4 marks)

Mark scheme · 7.10D Iteration: Fluency and Exam Drill

Question 1

- **B1** $f(1) = -2$ and $f(2) = 3$ so signs differ, hence a root in (1, 2) cao
- **Answer: $f(1) = 1 - 2 - 1 = -2$, $f(2) = 8 - 4 - 1 = 3$. Since $f(1) < 0$ and $f(2) > 0$ there is a root between 1 and 2 (by intermediate value theorem).**

Question 2

- **M1** substitute $x_0 = 2$ into $x_{n+1} = (x_n + 3/x_n)/2$ and calculate x_1
- **M1** use x_1 to calculate x_2 with the same formula
- **A1** $x_1 = 1.750$, $x_2 = 1.732$ cao
- **Answer: $x_1 = (2 + 3/2)/2 = 1.75$ -> 1.750; $x_2 = (1.75 + 3/1.75)/2$ approx 1.732142857 -> 1.732**

Question 3

- **M1** calculate $x_1 = (1.5 + 2/1.5)/2$ correctly
- **M1** calculate x_2 using x_1
- **A1** $x_2 = 1.414216$ (awrt) cao
- **Answer: $x_1 = (1.5 + 1.3333333)/2 = 1.4166667$; x_2 approx $(1.4166667 + 2/1.4166667)/2$ approx 1.4142157 -> 1.414216**

Question 4

- **B1** $f(0) = 1$ and $f(1) = -1$ so signs differ, hence a root in (0,1) cao
- **Answer: $f(0) = 0 - 0 + 1 = 1$, $f(1) = 1 - 3 + 1 = -1$. Since $f(0) > 0$ and $f(1) < 0$ there is a root between 0 and 1.**

Question 5

- **M1** calculate $x_1 = (10 - 1.5^3)/4 = 1.65625$
- **M1** calculate $x_2 = (10 - x_1^3)/4 = 1.36415863$ (awrt)
- **B1** conclude that the sequence does not appear to be converging (it oscillates) oe
- **Answer: $x_1 = (10 - 1.5^3)/4 = 6.625/4 = 1.65625$; $x_2 = (10 - 1.65625^3)/4$ approx 1.36415863. The values jump (1.5 -> 1.65625 -> 1.36416), so the sequence appears to oscillate and not settle (not converging).**

Question 6

- **M1** calculate $x_1 = \cos(1)$ and $x_2 = \cos(x_1)$
- **M1** calculate $x_3 = \cos(x_2)$
- **M1** give x_2 and x_3 to 3 dp correctly
- **A1** state the sequence appears to approach 0.739 (awrt) cao
- **Answer: $x_1 = \cos(1)$ approx 0.5403023059; $x_2 = \cos(0.5403023059)$ approx 0.8575532158 -> 0.858; $x_3 = \cos(0.8575532158)$ approx 0.6542897905 -> 0.654. The sequence appears to approach approximately 0.739 (the fixed point of cos x).**

Question 7

- **M1** state Newton formula $x_{n+1} = x_n - f(x_n)/f'(x_n)$ with $f' = 3x^2 - 1$
- **M1** calculate $x_1 = 1.5 - (-0.125)/5.75 = 35/23 = 1.5217391304$
- **M1** calculate x_2 using x_1 (use $f(x_1) = 26/12167$ and $f'(x_1) = 3146/529$ or equivalent)
- **A1** $x_2 = 1.5214$ (to 4 dp) cao
- **Answer: $x_1 = 35/23$ approx 1.5217391304. $f(x_1) = 26/12167$, $f'(x_1)=3146/529$, so x_2 approx 1.52137953 -> 1.5214 (4 dp).**

Question 8

- **M1** calculate $x_1 = 6/(1.5 + 1) = 6/2.5 = 2.4$
- **M1** calculate $x_2 = 6/(2.4 + 1) = 6/3.4 = 1.764705882$
- **A1** $x_2 = 1.765$ (awrt); values appear to be moving toward the positive root $x = 2$ (comment) oe
- **Answer: $x_1 = 2.4$; x_2 approx 1.764706 -> 1.765. The iterations move around the true positive root 2 and appear to be approaching it (oscillating but converging).**

Question 9

- **M1** evaluate f(2) = 8 - 4 - 5 = -1 and f(3) = 27 - 6 - 5 = 16
- **M1** calculate $x_1 = (5 + 2*2.5)^{1/3} = (10)^{1/3} = 10^{1/3}$ approx 2.154435
- **M1** calculate $x_2 = (5 + 2*x_1)^{1/3}$ approx $(5 + 4.30887)^{1/3} = 9.30887^{1/3}$ approx 2.104
- **A1** conclude whether iteration appears to converge (values move toward ~2.094...) cao
- **Answer: f(2) = -1, f(3) = 16 so a root lies between 2 and 3. $x_1 = 10^{1/3}$ approx 2.154435; x_2 approx $9.30887^{1/3}$ approx 2.104. The values move toward about 2.094; the rearrangement appears to produce convergence.**

Question 10

- **M1** state Newton formula x_{n+1} = $x_n - (x_n^3 - 7)/(3x_n^2)$
- **M1** calculate x_1 = 2 - 1/12 = 1.9166666667
- **M1** calculate x_2 using x_1: x_2 approx 1.9129385
- **A1** x_2 = 1.9129 (to 4 dp) cao
- **Answer: x_1 = 2 - (8 - 7)/12 = 1.9166666667. x_2 approx 1.9166666667 - $(1.9166666667^3 - 7)/(3*1.9166666667^2)$ approx 1.9129385 -> 1.9129 (4 dp).**