HIGHER

8.10 Vectors Proof Questions: 2-hour Pack

Total Marks

EDEXCEL 1MA1 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 OAB is a triangle. OA = a and OB = b. P is the point on AB such that AP:PB = 3:2. Find OP in terms of a and b. Give your answer in its simplest form.

(Total for Question 1 is 3 marks)

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2 OAB is a triangle. OA = a and OB = b. P is the point on OA such that OP:PA = 3:5 and Q is the point on OB such that OQ:QB = 3:5. Show that PQ is parallel to AB and state the value of k in PQ = k AB.

(Total for Question 2 is 4 marks)

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3 OAB is a triangle. OA = a and OB = b. M is the midpoint of AB. N is the point on OB such that ON:NB = 1:4. Find MN in terms of a and b.

(Total for Question 3 is 3 marks)

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4 OP = 6a + 4b, OQ = -2b and OR = -6a - 8b. Show that P, Q and R lie on a straight line.

(Total for Question 4 is 4 marks)

---

5 OAB is a triangle. OA = a and OB = b. P is the point on OA such that OP:PA = 3:2. Q is the point on OB such that OQ:QB = 2:1. Find PQ in terms of a and b.

(Total for Question 5 is 2 marks)

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6 OAB is a triangle. OA = a and OB = b. P is the point on OA such that OP:PA = 5:3 and Q is the point on OB such that OQ:QB = 5:3. Show that PQ is parallel to AB and state the value of k in PQ = k AB.

(Total for Question 6 is 4 marks)

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7 OAB is a triangle. OA = a and OB = b. M is the midpoint of AB. N is the point on OB such that ON:NB = 1:1. Find MN in terms of a and b.

(Total for Question 7 is 3 marks)

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8 OP = 4a + 4b, OQ = -a + 2b and OR = 9a + 6b. Show that P, Q and R lie on a straight line.

(Total for Question 8 is 4 marks)

---

9 OAB is a triangle. OA = a and OB = b. P is the point on OA such that OP:PA = 1:2. Q is the point on OB such that OQ:QB = 1:1. Find PQ in terms of a and b.

(Total for Question 9 is 2 marks)

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10 OAB is a triangle. OA = a and OB = b. P is the point on OA such that OP:PA = 1:3 and Q is the point on OB such that OQ:QB = 1:3. Show that PQ is parallel to AB and state the value of k in PQ = k AB.

(Total for Question 10 is 4 marks)

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11 OAB is a triangle. OA = a and OB = b. P is the point on AB such that AP:PB = 2:5. Find OP in terms of a and b. Give your answer in its simplest form.

(Total for Question 11 is 3 marks)

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12 OABC is a parallelogram. OA = a and OC = b. M is the point on AB such that AM:MB = 3:1. Find CM in terms of a and b.

(Total for Question 12 is 3 marks)

---

13 OAB is a triangle. OA = a and OB = b. P is the point on OA such that OP:PA = 2:1 and Q is the point on OB such that OQ:QB = 2:1. Show that PQ is parallel to AB and state the value of k in PQ = k AB.

(Total for Question 13 is 4 marks)

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14 OAB is a triangle. OA = a and OB = b. P is the point on AB such that AP:PB = 3:4. Find OP in terms of a and b. Give your answer in its simplest form.

(Total for Question 14 is 3 marks)

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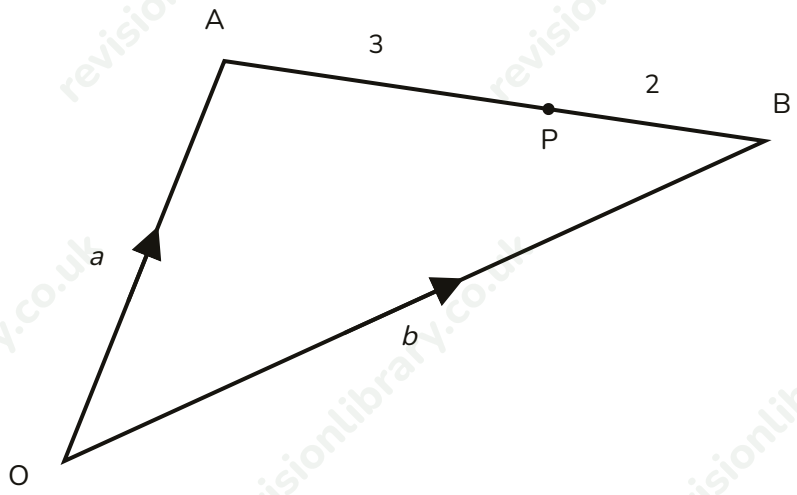
15 OABC is a parallelogram. OA = a and OC = b. M is the point on AB such that AM:MB = 4:3. Find CM in terms of a and b.

(Total for Question 15 is 3 marks)

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16 OAB is a triangle with OA = a and OB = b. P is the point on AB such that AP:PB = 3:2.

Diagram NOT accurately drawn



(a) Find AB in terms of a and b. **(1)**

.....

(b) Find AP in terms of a and b. **(1)**

.....

(c) Show that OP = (2/5)a + (3/5)b. **(2)**

(d) Given that OQ = 2a + 3b, show that O, P and Q lie on a straight line, and find the ratio OP:PQ. **(3)**

(Total for Question 16 is 7 marks)

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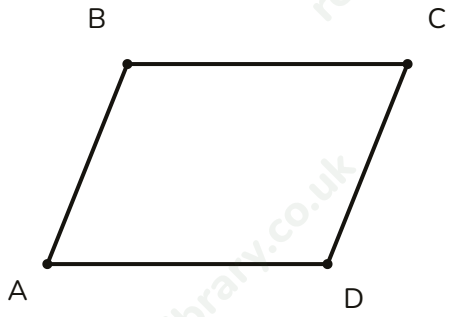
17 O is the origin. The position vectors of points P, Q and R are OP = 2a - b, OQ = 6a + kb and OR = a + 2b, where a and b are non-parallel vectors and k is a constant.
Given that P, Q and R lie on a straight line, find the value of k.

(Total for Question 17 is 4 marks)

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18 The position vectors of points A, B, C and D, relative to origin O, are OA = a, OB = a + 2b, OC = 2a + 2b and OD = 2a.

Diagram NOT accurately drawn



(a) Find AB in terms of b. **(1)**

.....

(b) Find DC in terms of b. **(1)**

.....

(c) Find AD and BC, and hence prove that ABCD is a parallelogram. **(3)**

(Total for Question 18 is 5 marks)

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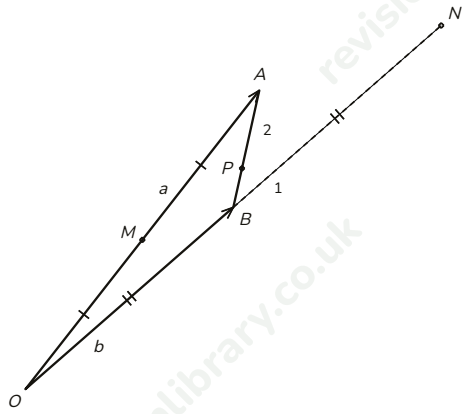
19 Let OA = a, OB = b and OC = 5a - 4b. Show that A, B and C are collinear and state where C lies in relation to A and B.

(Total for Question 19 is 3 marks)

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20 OAB is a triangle with OA = a and OB = b. M is the midpoint of OA. N is the point on OB extended beyond B such that OB = BN. P is the point on AB such that AP:PB = 2:1.

Diagram NOT accurately drawn



(a) Find OM in terms of a. **(1)**

.....

(b) Find ON in terms of b. **(1)**

.....

(c) Find OP in terms of a and b. **(2)**

.....

(d) Prove that M, P and N lie on a straight line. **(3)**

(e) Find the ratio MP:PN. **(1)**

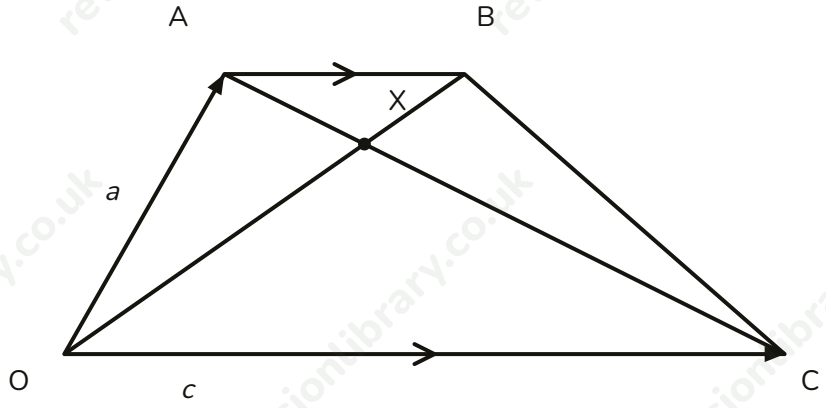
.....

(Total for Question 20 is 8 marks)

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21 OABC is a trapezium in which OC is parallel to AB, and OC = 3AB. OA = a and OC = c.

Diagram NOT accurately drawn



(a) Find AB in terms of c. **(1)**

.....

(b) Find OB in terms of a and c. **(1)**

.....

(c) Find AC in terms of a and c. **(1)**

.....

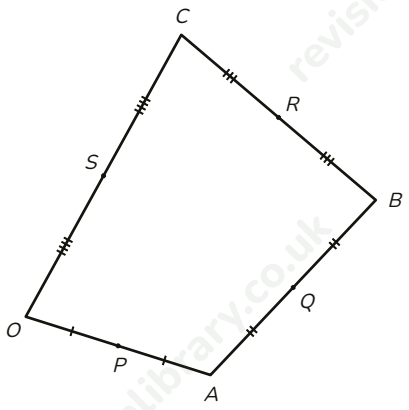
(d) The diagonals OB and AC intersect at X. Prove that OX:XB = 3:1. **(4)**

(Total for Question 21 is 7 marks)

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22 O, A, B and C are four points such that OA = a, OB = b and OC = c. P, Q, R and S are the midpoints of OA, AB, BC and CO respectively.

Diagram NOT accurately drawn



(a) Find OP in terms of a. **(1)**

.....

(b) Find OQ in terms of a and b. **(1)**

.....

(c) Show that PQ = (1/2)b. **(2)**

(d) Find OS and OR, and hence show that SR = (1/2)b. **(2)**

(e) Hence prove that PQRS is a parallelogram. **(1)**

(Total for Question 22 is 7 marks)

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Mark scheme · 8.10 Vectors Proof Questions: 2-hour Pack

Question 1

AB = -a + b	**M1**
AP = 3/5 of AB	**M1**
OP = (2/5)a + (3/5)b oe	**A1**

Answer: OP = (2/5)a + (3/5)b

Question 2

OP = 3/8a and OQ = 3/8b	**M1**
PQ = 3/8b - 3/8a = 3/8(b - a)	**M1**
AB = b - a	**M1**
PQ = (3/8)AB, so PQ is parallel to AB	**A1**

Answer: PQ = (3/8)AB, so PQ is parallel to AB

Question 3

OM = 1/2(a + b)	**M1**
ON = 1/5b	**M1**
MN = -(1/2)a - (3/10)b oe	**A1**

Answer: MN = -(1/2)a - (3/10)b

Question 4

PQ = OQ - OP = -6a - 6b	**M1**
PR = OR - OP = -12a - 12b	**M1**
PR = 2PQ	**A1**
common point P and conclusion that P, Q and R are collinear	**B1**

Answer: PR = 2PQ, so P, Q and R are collinear

Question 5

PQ = PO + OQ or OQ - OP	**M1**
-(3/5)a + (2/3)b oe	**A1**

Answer: PQ = -(3/5)a + (2/3)b

Question 6

OP = 5/8a and OQ = 5/8b	**M1**
PQ = 5/8b - 5/8a = 5/8(b - a)	**M1**
AB = b - a	**M1**
PQ = (5/8)AB, so PQ is parallel to AB	**A1**

Answer: PQ = (5/8)AB, so PQ is parallel to AB

Question 7

OM = 1/2(a + b)	**M1**
ON = 1/2b	**M1**
MN = -(1/2)a oe	**A1**

Answer: MN = -(1/2)a

Question 8

PQ = OQ - OP = -5a - 2b	**M1**
PR = OR - OP = 5a + 2b	**M1**
PR = -PQ	**A1**
common point P and conclusion that P, Q and R are collinear	**B1**

Answer: PR = -PQ, so P, Q and R are collinear

Question 9

PQ = PO + OQ or OQ - OP	**M1**
-(1/3)a + (1/2)b oe	**A1**

Answer: PQ = -(1/3)a + (1/2)b

Question 10

OP = 1/4a and OQ = 1/4b	**M1**
PQ = 1/4b - 1/4a = 1/4(b - a)	**M1**
AB = b - a	**M1**
PQ = (1/4)AB, so PQ is parallel to AB	**A1**

Answer: PQ = (1/4)AB, so PQ is parallel to AB

Question 11

AB = -a + b	**M1**
AP = 2/7 of AB	**M1**
OP = (5/7)a + (2/7)b oe	**A1**

Answer: OP = (5/7)a + (2/7)b

Question 12

AB = OC = b so OB = a + b or OM = a + AM	**M1**
AM = 3/4b and CM = OM - OC	**M1**
CM = a - (1/4)b oe	**A1**

Answer: CM = a - (1/4)b

Question 13

OP = 2/3a and OQ = 2/3b	**M1**
PQ = 2/3b - 2/3a = 2/3(b - a)	**M1**
AB = b - a	**M1**
PQ = (2/3)AB, so PQ is parallel to AB	**A1**

Answer: PQ = (2/3)AB, so PQ is parallel to AB

Question 14

AB = -a + b	**M1**
AP = 3/7 of AB	**M1**
OP = (4/7)a + (3/7)b oe	**A1**

Answer: OP = (4/7)a + (3/7)b

Question 15

AB = OC = b so OB = a + b or OM = a + AM	**M1**
AM = 4/7b and CM = OM - OC	**M1**
CM = a - (3/7)b oe	**A1**

Answer: CM = a - (3/7)b

Question 16

(a) b - a (oe) (cao)	**B1**

(a) Answer: AB = b - a

(b) (3/5)(b - a) (oe) (ft their part (a))	**B1**

(b) Answer: AP = (3/5)(b - a)

(c) OP = OA + AP = a + (3/5)(b - a) (ft their part (b))	**M1**
(c) correctly simplifies to (2/5)a + (3/5)b (cso)	**A1**

(c) Answer: OP = (2/5)a + (3/5)b

(d) recognises OP = (1/5)(2a + 3b) (ft their part (c))	**M1**
(d) OP = (1/5)OQ, and since O, P and Q share the common point O with OP a scalar multiple of OQ, O, P and Q are collinear (cso)	**A1**
(d) OP:PQ = 1:4 (cao)	**A1**

(d) Answer: O, P, Q are collinear; OP:PQ = 1:4

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Question 17

finds PQ = OQ - OP = 4a + (k+1)b (oe) **M1**

finds PR = OR - OP = -a + 3b (oe) **M1**

sets PQ = m x PR and equates coefficients of a: 4 = -m, so m = -4 **M1**

k = -13 (cao) **A1**

Answer: k = -13

Question 18

(a) 2b (cao) **B1**

(a) Answer: AB = 2b

(b) 2b (cao) **B1**

(b) Answer: DC = 2b

(c) AD = OD - OA = a (oe) **M1**

(c) BC = OC - OB = a (oe) **M1**

(c) AD = BC and AB = DC, so both pairs of opposite sides are equal and parallel, hence ABCD is a parallelogram (cso) **A1**

(c) Answer: AD = BC = a and AB = DC = 2b, so ABCD is a parallelogram

Question 19

compute OC - OA = 4a - 4b = 4(a - b) = -4(b - a) oe **M1**

state A, B and C are collinear with OC - OA = -4(OB - OA) cao **A1**

state C lies on the line AB produced beyond A (k = -4) cao **A1**

Answer: Collinear; OC - OA = -4(OB - OA) so C is on line AB produced beyond A (scalar k = -4).

Question 20

(a) (1/2)a (oe) (cao) **B1**

(a) Answer: OM = (1/2)a

(b) 2b (oe) (cao) **B1**

(b) Answer: ON = 2b

(c) AP = (2/3)(b - a), and OP = OA + AP **M1**

(c) (1/3)a + (2/3)b (oe) (cao) **A1**

(c) Answer: OP = (1/3)a + (2/3)b

(d) finds MP = OP - OM = -(1/6)a + (2/3)b (ft their parts (a) and (c)) **M1**

(d) finds MN = ON - OM = -(1/2)a + 2b (ft their parts (a) and (b)) **M1**

(d) MN = 3 x MP (since -(1/2) = 3 x -(1/6) and 2 = 3 x (2/3)), so MN is a scalar multiple of MP, and since M is a common point, M, P and N are collinear (cso) **A1**

(d) Answer: M, P and N are collinear (MN = 3MP)

(e) 1:2 (cao) (ft their part (d)) **B1**

(e) Answer: MP:PN = 1:2

Question 21

(a) (1/3)c (oe) (cao) **B1**

(a) Answer: AB = (1/3)c

(b) a + (1/3)c (oe) (ft their part (a)) **B1**

(b) Answer: OB = a + (1/3)c

(c) c - a (oe) (cao) **B1**

(c) Answer: AC = c - a

(d) X lies on OB, so OX = m(a + (1/3)c) for some scalar m (ft their part (b)) **M1**

(d) X lies on AC, so OX = OA + h.AC = (1-h)a + hc for some scalar h (ft their part (c)) **M1**

(d) equates coefficients of a and c: m = 1-h and m/3 = h; substitutes to get m = 1 - m/3, leading to m = 3/4 and h = 1/4 **dM1**

(d) OX = (3/4)OB, so OX:XB = 3:1 (cso) **A1**

(d) Answer: OX:XB = 3:1

Question 22

(a) (1/2)a (oe) (cao) **B1**

(a) Answer: OP = (1/2)a

(b) (1/2)(a + b) (oe) (cao) **B1**

(b) Answer: OQ = (1/2)(a + b)

(c) PQ = OQ - OP = (1/2)(a+b) - (1/2)a (ft their parts (a) and (b)) **M1**

(c) correctly simplifies to (1/2)b (cso) **A1**

(c) Answer: PQ = (1/2)b

(d) OS = (1/2)c (S is the midpoint of CO) and OR = (1/2)(b+c) (R is the midpoint of BC) **M1**

(d) SR = OR - OS = (1/2)b (cso) **A1**

(d) Answer: OS = (1/2)c, OR = (1/2)(b+c), SR = (1/2)b

(e) PQ = SR = (1/2)b, so PQ is parallel and equal to SR, therefore PQRS is a parallelogram (cso, ft their parts (c) and (d)) **B1**

(e) Answer: PQRS is a parallelogram, since PQ = SR = (1/2)b

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