



8.10D Vectors Proof Questions: Fluency and Exam Drill

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 50 minutes

Total Marks

Name: _____ Date: ____ / ____ / ____

Answer ALL questions. Show all your working.

- 1 Vectors $OA = \langle 3, 2 \rangle$ and $OB = \langle -1, 4 \rangle$. Work out the vector AB .

(Total for Question 1 is 1 mark)

- 2 $OA = \langle 1, 2 \rangle$, $OB = \langle 4, 5 \rangle$. Point P has position vector $OP = (2/3)OA + (1/3)OB$.
Show that P lies on the line AB and find the ratio $AP:PB$.

(Total for Question 2 is 3 marks)

- 3 $OA = \langle 1, 0 \rangle$, $OB = \langle 4, 3 \rangle$, $OC = \langle 7, 6 \rangle$. Show that A , B and C are collinear.

(Total for Question 3 is 2 marks)

- 4 $OA = \langle 2, 1 \rangle$, $OC = \langle -4, 7 \rangle$. Show that B , with $OB = \langle -1, 4 \rangle$, is the midpoint of AC .

(Total for Question 4 is 2 marks)

- 5 $OA = \langle 2, 0 \rangle$, $OB = \langle 8, 6 \rangle$, $OC = \langle 5, 3 \rangle$. Prove that A , B and C are collinear.

(Total for Question 5 is 2 marks)

- 6** $OA = \langle 2, 1 \rangle$, $OB = \langle 12, 11 \rangle$. Point P divides AB internally so that $AP:PB = 3:2$. Find the position vector OP and show working.

(Total for Question 6 is 3 marks)

- 7** $OA = \langle 1, 2 \rangle$, $OB = \langle 4, 3 \rangle$, $OC = \langle 7, 4 \rangle$. Prove that A, B and C are collinear by showing AC is a multiple of AB.

(Total for Question 7 is 2 marks)

- 8** $OA = \langle 0, 0 \rangle$, $OB = \langle 2, 1 \rangle$, $OC = \langle 3, 4 \rangle$. Show that A, B and C are not collinear.

(Total for Question 8 is 2 marks)

- 9** Let $OA = a$ and $OB = b$. Point C has position vector $OC = \frac{1}{3}a + \frac{2}{3}b$. Show that C lies on AB and find AC:CB.

(Total for Question 9 is 3 marks)

- 10** $OA = \langle 2, 3 \rangle$, $OB = \langle 8, 7 \rangle$, $OC = \langle -4, -1 \rangle$. Show that A is the midpoint of BC.

(Total for Question 10 is 3 marks)

- 11** Let $OA = a$, $OB = b$ and $OC = 5a - 4b$. Show that A, B and C are collinear and state where C lies in relation to A and B.

(Total for Question 11 is 3 marks)

- 12** $OA = a$ and $OB = b$. D is a point on AB such that $AD = 2 DB$. Show that $OD = \frac{1}{3}a + \frac{2}{3}b$.

(Total for Question 12 is 3 marks)

Mark scheme · 8.10D Vectors Proof Questions: Fluency and Exam Drill

Question 1

- **B1** $\mathbf{AB} = \langle -4, 2 \rangle$ **cao**
- **Answer:** $\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = \langle -4, 2 \rangle$.

Question 2

- **M1** calculate $\mathbf{OP} = (2/3)\langle 1, 2 \rangle + (1/3)\langle 4, 5 \rangle = \langle 2, 3 \rangle$ **oe**
- **M1** write $\mathbf{OP} = \mathbf{OA} + t(\mathbf{OB} - \mathbf{OA})$ and solve $1 + 3t = 2$, $2 + 3t = 3$ to get $t = 1/3$ **oe**
- **A1** $\mathbf{AP}:\mathbf{PB} = 1:2$ **cao**
- **Answer:** **P lies on AB with $\mathbf{AP}:\mathbf{PB} = 1:2$. ($\mathbf{OP} = \langle 2, 3 \rangle$, $t = 1/3$ so $\mathbf{AP} = 1/3 \mathbf{AB}$.)**

Question 3

- **M1** calculate $\mathbf{OC} - \mathbf{OA} = \langle 6, 6 \rangle$ and $\mathbf{OB} - \mathbf{OA} = \langle 3, 3 \rangle$ and show $\mathbf{OC} - \mathbf{OA} = 2(\mathbf{OB} - \mathbf{OA})$ **oe**
- **A1** state A, B and C are collinear **cao**
- **Answer:** **Collinear, since $\mathbf{OC} - \mathbf{OA} = 2(\mathbf{OB} - \mathbf{OA})$.**

Question 4

- **M1** calculate $(\mathbf{OA} + \mathbf{OC})/2 = (\langle 2, 1 \rangle + \langle -4, 7 \rangle)/2 = \langle -1, 4 \rangle$ **oe**
- **A1** state $\mathbf{OB} = (\mathbf{OA} + \mathbf{OC})/2$ so B is midpoint of AC **cao**
- **Answer:** **B is the midpoint of AC because $\mathbf{OB} = (\mathbf{OA} + \mathbf{OC})/2 = \langle -1, 4 \rangle$.**

Question 5

- **M1** calculate $\mathbf{AB} = \langle 6, 6 \rangle$ and $\mathbf{AC} = \langle 3, 3 \rangle$ and show $\mathbf{AC} = 1/2 \mathbf{AB}$ **oe**
- **A1** state A, B and C are collinear **cao**
- **Answer:** **Collinear, since $\mathbf{AC} = 1/2 \mathbf{AB}$.**

Question 6

- **M1** use $\mathbf{OP} = (n\mathbf{OA} + m\mathbf{OB})/(m+n)$ with $\mathbf{AP}:\mathbf{PB} = m:n$ so $\mathbf{OP} = (2\mathbf{OA} + 3\mathbf{OB})/5$ **oe**
- **M1** calculate $(2\langle 2, 1 \rangle + 3\langle 12, 11 \rangle)/5 = \langle 8, 7 \rangle$ **oe**
- **A1** final $\mathbf{OP} = \langle 8, 7 \rangle$ **cao**
- **Answer:** **$\mathbf{OP} = \langle 8, 7 \rangle$. (Using $\mathbf{OP} = (2\mathbf{OA} + 3\mathbf{OB})/5$ gives $\langle 8, 7 \rangle$.)**

Question 7

- **M1** calculate $\mathbf{AB} = \langle 3, 1 \rangle$, $\mathbf{AC} = \langle 6, 2 \rangle$ and show $\mathbf{AC} = 2 \mathbf{AB}$ **oe**
- **A1** state A, B and C are collinear **cao**
- **Answer:** **Collinear because $\mathbf{AC} = 2 \mathbf{AB}$.**

Question 8

- **M1** calculate $\mathbf{AB} = \langle 2, 1 \rangle$ and $\mathbf{AC} = \langle 3, 4 \rangle$ and show there is no scalar k with $\langle 3, 4 \rangle = k\langle 2, 1 \rangle$ ($3/2 \neq 4/1$) **oe**
- **A1** state A, B and C are not collinear **cao**
- **Answer:** **Not collinear because $\mathbf{AC}$ is not a scalar multiple of $\mathbf{AB}$ ($3/2 \neq 4/1$).**

Question 9

- **M1** calculate $\mathbf{OC} - \mathbf{OA} = -2/3 \mathbf{a} + 2/3 \mathbf{b} = 2/3 (\mathbf{b} - \mathbf{a})$ **oe**
- **M1** identify $k = 2/3$ so $\mathbf{AC} = (2/3) \mathbf{AB}$ **oe**
- **A1** $\mathbf{AC}:\mathbf{CB} = 2:1$ **cao**
- **Answer:** **C lies on AB and $\mathbf{AC}:\mathbf{CB} = 2:1$ (since $\mathbf{OC} - \mathbf{OA} = 2/3(\mathbf{b} - \mathbf{a})$).**

Question 10

- **M1** calculate $(\mathbf{OB} + \mathbf{OC})/2 = (<8,7> + <-4,-1>)/2 = <2,3>$ oe
- **M1** recognise this equals $\mathbf{OA}$ so $\mathbf{OA} = (\mathbf{OB} + \mathbf{OC})/2$ oe
- **A1** state A is midpoint of BC cao
- **Answer: A is midpoint of BC because $\mathbf{OA} = (\mathbf{OB} + \mathbf{OC})/2 = <2,3>$.**

Question 11

- **M1** compute $\mathbf{OC} - \mathbf{OA} = 4\mathbf{a} - 4\mathbf{b} = 4(\mathbf{a} - \mathbf{b}) = -4(\mathbf{b} - \mathbf{a})$ oe
- **A1** state A, B and C are collinear with $\mathbf{OC} - \mathbf{OA} = -4(\mathbf{OB} - \mathbf{OA})$ cao
- **A1** state C lies on the line AB produced beyond A ($k = -4$) cao
- **Answer: Collinear; $\mathbf{OC} - \mathbf{OA} = -4(\mathbf{OB} - \mathbf{OA})$ so C is on line AB produced beyond A (scalar $k = -4$).**

Question 12

- **M1** use $\mathbf{AD} = 2/3 \mathbf{AB}$ so vector $\mathbf{AD} = 2/3(\mathbf{b} - \mathbf{a})$ oe
- **M1** write $\mathbf{OD} = \mathbf{OA} + \mathbf{AD} = \mathbf{a} + 2/3(\mathbf{b} - \mathbf{a})$ and simplify oe
- **A1** $\mathbf{OD} = 1/3 \mathbf{a} + 2/3 \mathbf{b}$ cso
- **Answer: $\mathbf{OD} = 1/3 \mathbf{a} + 2/3 \mathbf{b}$. (Using $\mathbf{AD} = 2/3 \mathbf{AB}$ gives $\mathbf{OD} = \mathbf{a} + 2/3(\mathbf{b} - \mathbf{a}) = 1/3 \mathbf{a} + 2/3 \mathbf{b}$.)**