HIGHER

8.3 Proof: 1-hour Pack

EDEXCEL 1MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

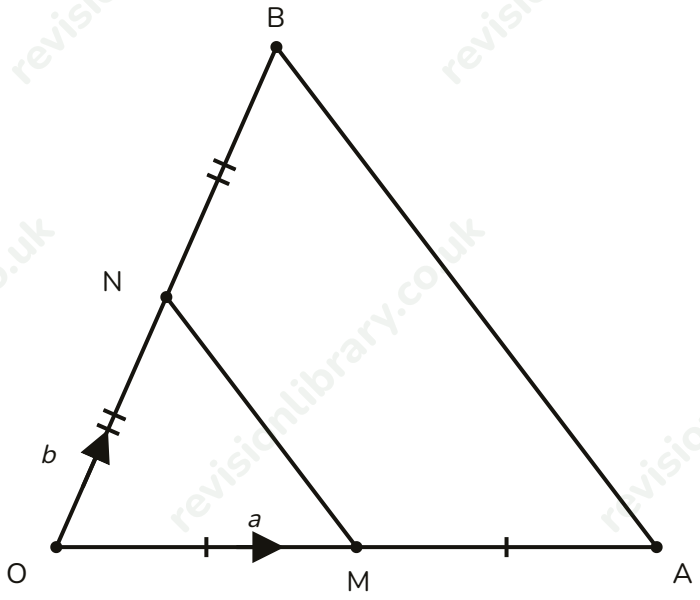
1 Prove algebraically that the difference between the squares of any two consecutive even numbers is always a multiple of 4.

(Total for Question 1 is 3 marks)

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2 OAB is a triangle, where OA = vector a and OB = vector b. M is the midpoint of OA and N is the midpoint of OB. Prove that MN is parallel to AB, and that MN = (1/2) AB.

Diagram NOT accurately drawn



(Total for Question 2 is 5 marks)

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3 Prove algebraically that the sum of any four consecutive integers is always even, but is never a multiple of 4.

(Total for Question 3 is 4 marks)

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4 n is an integer. Show that the sum of 5 consecutive integers, n, n + 1, ..., n + 4, is a multiple of 5.

(Total for Question 4 is 3 marks)

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5 n is an integer. Show that $(3n + 3)^2 - (3n - 3)^2$ is a multiple of 36.

(Total for Question 5 is 3 marks)

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6 n is an integer. Show that $(n + 15)^2 - (n - 15)^2$ is a multiple of 60.

(Total for Question 6 is 3 marks)

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7 n is an integer. Show that $(2n + 12)^2 - 144$ is a multiple of 4.

(Total for Question 7 is 3 marks)

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8 n is an integer. Show that $(2n + 7)^2 + (2n + 9)^2$ is always even.

(Total for Question 8 is 3 marks)

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9 n is an integer. Show that $(2n + 4)^2 - (2n - 4)^2$ is a multiple of 32.

(Total for Question 9 is 3 marks)

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10 n is an integer. Show that the sum of 7 consecutive integers, n, n + 1, ..., n + 6, is a multiple of 7.

(Total for Question 10 is 3 marks)

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11 n is an integer. Show that $(4n + 7)^2 - (4n - 7)^2$ is a multiple of 112.

(Total for Question 11 is 3 marks)

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12 n is an integer. Show that (n + 3)(n + 9) - n(n + 12) does not depend on the value of n.

(Total for Question 12 is 3 marks)

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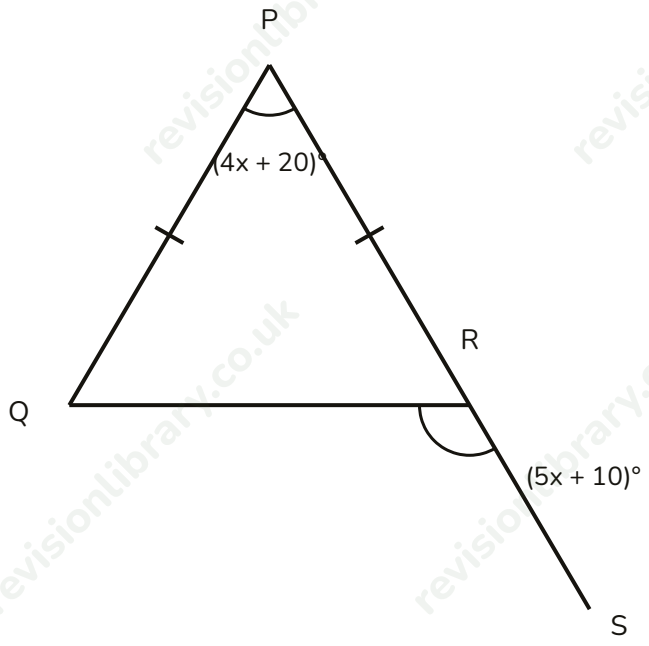
13 Chris says that $n^2 - n + 17$ is a prime number for every positive whole number n. Show that Chris is wrong by finding the smallest value of n that gives a number that is not prime.

(Total for Question 13 is 2 marks)

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14 The diagram shows triangle PQR, where PQ = PR. Angle QPR = (4x + 20) degrees. S is a point on PR extended beyond R, so that angle QRS = (5x + 10) degrees. Prove that x = 30.

Diagram NOT accurately drawn



(Total for Question 14 is 4 marks)

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Mark scheme · 8.3 Proof: 1-hour Pack

Question 1

Consecutive even numbers written as 2n and 2n + 2 (oe)	**M1**
$(2n+2)^2 - (2n)^2$ expanded and simplified to 8n + 4 (oe)	**M1**
Factorised to 4(2n+1) with conclusion that this is a multiple of 4 (cso)	**A1**

Answer: $(2n+2)^2 - (2n)^2 = 8n + 4 = 4(2n+1)$, a multiple of 4.

Question 2

OM = (1/2)a stated	**M1**
ON = (1/2)b stated	**M1**
MN = ON - OM = (1/2)b - (1/2)a (oe)	**M1**
AB = OB - OA = b - a, so MN = (1/2)(b - a) = (1/2)AB (oe)	**A1**
Conclusion: since MN is a scalar multiple of AB, MN is parallel to AB, and MN = (1/2)AB (cso)	**B1**

Answer: MN = (1/2)(b - a) = (1/2)AB, so MN is parallel to AB and half its length.

Question 3

Four consecutive integers n, n+1, n+2, n+3 summed and simplified to 4n + 6	**M1**
Factorised to 2(2n + 3), showing the sum is even (has factor 2)	**A1**
2n + 3 identified as odd (2n is even, plus 3 is odd)	**M1**
Conclusion: 2(2n+3) is even but, since 2n+3 is odd, it cannot be a multiple of 4 (cso)	**A1**

Answer: n + (n+1) + (n+2) + (n+3) = 4n + 6 = 2(2n+3). This is even, but since 2n+3 is odd, the sum is never a multiple of 4.

Question 4

expands brackets correctly	**M1**
simplifies correctly	**M1**
5(n + 2) with a statement that a multiple of 5	**A1**

Answer: 5(n + 2), which is a multiple of 5

Question 5

expands brackets correctly	**M1**
simplifies correctly	**M1**
36n with a statement that a multiple of 36	**A1**

Answer: 36n, which is a multiple of 36

Question 6

expands brackets correctly	**M1**
simplifies correctly	**M1**
60n with a statement that a multiple of 60	**A1**

Answer: 60n, which is a multiple of 60

Question 7

expands brackets correctly	**M1**
simplifies correctly	**M1**
4n(n + 12) with a statement that a multiple of 4	**A1**

Answer: 4n(n + 12), which is a multiple of 4

Question 8

expands brackets correctly	**M1**
simplifies correctly	**M1**
$2(4n^2 + 32n + 65)$ with a statement that even	**A1**

Answer: $2(4n^2 + 32n + 65)$, which is even

Question 9

expands brackets correctly	**M1**
simplifies correctly	**M1**
32n with a statement that a multiple of 32	**A1**

Answer: 32n, which is a multiple of 32

Question 10

expands brackets correctly	**M1**
simplifies correctly	**M1**
7(n + 3) with a statement that a multiple of 7	**A1**

Answer: 7(n + 3), which is a multiple of 7

Question 11

expands brackets correctly	**M1**
simplifies correctly	**M1**
112n with a statement that a multiple of 112	**A1**

Answer: 112n, which is a multiple of 112

Question 12

expands brackets correctly	**M1**
simplifies correctly	**M1**
27 with a statement that a constant, independent of n	**A1**

Answer: 27, which is a constant, independent of n

Question 13

tests values of n in order or a counter example found	**M1**
n = 17 giving 289 with a factor shown	**A1**

Answer: n = 17: 289 = 17 x 17

Question 14

Base angles PQR = PRQ = (180 - (4x+20))/2 = 80 - 2x (oe), using the angle sum of a triangle and isosceles property	**M1**
Exterior angle QRS = 180 - (80 - 2x) = 100 + 2x (oe), using angles on a straight line	**M1**
Equation 100 + 2x = 5x + 10 formed and rearranged	**M1**
x = 30 (cso, answer given)	**A1**

Answer: x = 30

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