HIGHER

8.3 Proof: 30-minute Pack

EDEXCEL 1MA1 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Prove that $n^2 + n$ is even for any integer n.

(Total for Question 1 is 2 marks)

2 Prove that $n^3 - n$ is divisible by 6 for any integer n.

(Total for Question 2 is 3 marks)

3 Prove that n and n^2 have the same parity. That is, show n is even if and only if n^2 is even, and n is odd if and only if n^2 is odd.

(Total for Question 3 is 3 marks)

4 Prove that $n^3 + 5n$ is divisible by 6 for any integer n.

(Total for Question 4 is 4 marks)

© Revision Library | revisionlibrary.co.uk

5 Prove algebraically that the product of any two even numbers is always a multiple of 4.

(Total for Question 5 is 2 marks)

---

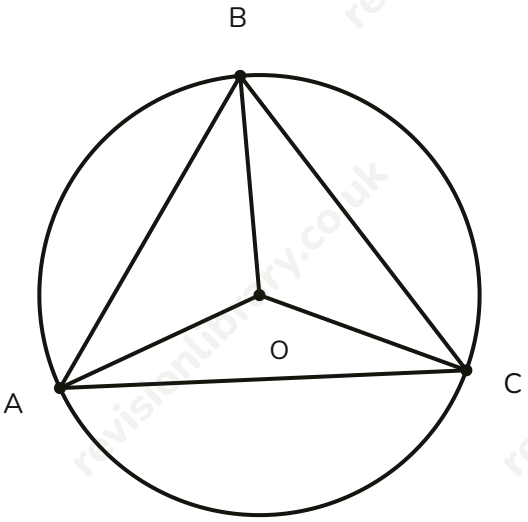
6 Let n be an odd integer. Prove that $n^2 - 1$ is divisible by 8.

(Total for Question 6 is 4 marks)

© Revision Library | revisionlibrary.co.uk

7 The diagram shows a circle with centre O. A, B and C are points on the circumference. Prove that angle AOC = 2 x angle ABC (the angle at the centre is twice the angle at the circumference subtended by the same arc).

Diagram NOT accurately drawn



(Total for Question 7 is 5 marks)

---

8 n is an integer. Show that $(n + 4)(n + 5) - n(n + 9)$ does not depend on the value of n.

(Total for Question 8 is 3 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 8.3 Proof: 30-minute Pack

Question 1

factorise to n(n + 1) or show product of two consecutive integers oe	**M1**
conclude n(n + 1) is even, so $n^2 + n$ is even cao	**A1**

Answer: n(n + 1) is even, so $n^2 + n$ is even (for all integers n).

Question 2

factorise to n(n - 1)(n + 1) oe	**M1**
state that among three consecutive integers one is even and one is a multiple of 3 (so product divisible by 2 and by 3)	**M1**
conclude $n^3 - n$ is divisible by 6 cao	**A1**

Answer: $n^3 - n = n(n - 1)(n + 1)$ is divisible by 6.

Question 3

show if n = 2k then $n^2 = 4k^2$ which is even (even case)	**M1**
show if n = 2k + 1 then $n^2 = 4k^2 + 4k + 1$ which is odd (odd case)	**M1**
conclude n and n^2 have the same parity cao	**A1**

Answer: If n = 2k then n^2 is even; if n = 2k + 1 then n^2 is odd. So n and n^2 have the same parity.

Question 4

rewrite $n^3 + 5n$ as $(n^3 - n) + 6n$	**M1**
show $n^3 - n = n(n - 1)(n + 1)$, which is divisible by 6 (product of three consecutive integers)	**M1**
state 6n is divisible by 6	**M1**
conclude $n^3 + 5n$ is divisible by 6 cao	**A1**

Answer: $n^3 + 5n = (n^3 - n) + 6n$, both parts divisible by 6, so $n^3 + 5n$ divisible by 6.

Question 5

Even numbers written as 2n and 2m, product formed as 2n x 2m = 4nm (oe)	**M1**
Conclusion that 4nm is a multiple of 4 for integers n and m (cso)	**A1**

Answer: 2n x 2m = 4nm, which is a multiple of 4 for all integers n and m.

Question 6

substitute n = 2k + 1 and expand: $n^2 - 1 = (2k + 1)^2 - 1$	**M1**
simplify to 4k(k + 1)	**M1**
state k(k + 1) is even (product of consecutive integers) so has factor 2	**M1**
conclude $n^2 - 1 = 4k(k + 1)$ is divisible by 8 cao	**A1**

Answer: If n = 2k + 1 then $n^2 - 1 = 4k(k + 1)$. Since k(k + 1) is even, $n^2 - 1$ is divisible by 8.

Question 7

OA = OB = OC (radii of the circle), so triangles OAB and OBC are isosceles	**B1**
Base angles labelled: angle OAB = angle OBA = x, and angle OCB = angle OBC = y	**B1**
Angle AOB = 180 - 2x and angle BOC = 180 - 2y (angle sum of a triangle = 180 degrees)	**M1**
Angle AOC = 360 - (180 - 2x) - (180 - 2y) = 2x + 2y (angles around a point sum to 360 degrees)	**dM1**
Angle ABC = x + y, so angle AOC = 2(x+y) = 2 x angle ABC (cso)	**A1**

Answer: Angle AOC = 2x + 2y = 2(x + y) = 2 x angle ABC.

Question 8

expands brackets correctly	**M1**
simplifies correctly	**M1**
20 with a statement that a constant, independent of n	**A1**

Answer: 20, which is a constant, independent of n

© Revision Library | revisionlibrary.co.uk