HIGHER

8.3 Proof: Test Pack

EDEXCEL 1MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Show that the difference of consecutive square numbers $(n + 1)^2 - n^2$ is odd for any integer n.

(Total for Question 1 is 2 marks)

2 Show that the sum of three consecutive integers $n + (n + 1) + (n + 2)$ is divisible by 3 for any integer n.

(Total for Question 2 is 3 marks)

3 Prove that the sum of three consecutive integers is always a multiple of 3.

(Total for Question 3 is 3 marks)

4 Prove that $(3 + \sqrt{2})(3 - \sqrt{2})$ is a rational number.

(Total for Question 4 is 3 marks)

© Revision Library | revisionlibrary.co.uk

5 Prove that the recurring decimal 0.181818... (0.18 recurring) is equal to 2/11.

(Total for Question 5 is 4 marks)

---

6 Kwame says, 'For every positive integer value of n, the expression $n^2 + n + 41$ gives a prime number.' Show, by finding a counter-example, that Kwame is not correct.

(Total for Question 6 is 3 marks)

---

7 Chris says that $n^2 - n + 11$ is a prime number for every positive whole number n. Show that Chris is wrong by finding the smallest value of n that gives a number that is not prime.

(Total for Question 7 is 2 marks)

---

8 n is an integer. Show that $(n + 8)^2 - (n - 8)^2$ is a multiple of 32.

(Total for Question 8 is 3 marks)

---

9 n is an integer. Show that the sum of 9 consecutive integers, n, n + 1, ..., n + 8, is a multiple of 9.

(Total for Question 9 is 3 marks)

© Revision Library | revisionlibrary.co.uk

10 n is an integer. Show that $(n + 1)^2 - (n - 1)^2$ is a multiple of 4.

(Total for Question 10 is 3 marks)

---

11 n is an integer. Show that $(2n + 4)^2 - 16$ is a multiple of 4.

(Total for Question 11 is 3 marks)

---

12 Stretch question. Prove that $n^5 - n$ is divisible by 30 for any integer n.

(Total for Question 12 is 5 marks)

© Revision Library | revisionlibrary.co.uk

13 Prove, by exhaustion, that $n^2 + n + 1$ is an odd number for every integer n from 1 to 5 inclusive.

(Total for Question 13 is 4 marks)

---

14 Prove, by completing the square, that $x^2 - 6x + 10 > 0$ for all real values of x.

(Total for Question 14 is 4 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 8.3 Proof: Test Pack

Question 1

calculate $(n + 1)^2 - n^2 = 2n + 1$	**M1**
state $2n + 1$ is odd, so the difference is odd cao	**A1**

Answer: $(n + 1)^2 - n^2 = 2n + 1$, which is odd for any integer n.

Question 2

sum the three terms to get $3n + 3$	**M1**
factorise to $3(n + 1)$	**M1**
conclude the sum is divisible by 3 cao	**A1**

Answer: $n + (n + 1) + (n + 2) = 3(n + 1)$, so divisible by 3.

Question 3

Consecutive integers written algebraically as n, n+1, n+2 (oe)	**M1**
Sum simplified to $3n + 3$ (oe)	**M1**
Factorised to $3(n+1)$ with conclusion that this is a multiple of 3 (cso)	**A1**

Answer: $n + (n+1) + (n+2) = 3n + 3 = 3(n+1)$, which is a multiple of 3 for any integer n.

Question 4

Expansion using difference of two squares: $3^2 - (\sqrt{2})^2$	**M1**
Simplified to $9 - 2 = 7$ (cao)	**A1**
Conclusion that 7 is a rational number (an integer, so rational), cso	**B1**

Answer: $(3+\sqrt{2})(3-\sqrt{2}) = 9 - 2 = 7$, which is rational.

Question 5

Let $x = 0.181818...$ and form $100x = 18.181818...$ (oe)	**M1**
Subtraction $100x - x = 18.181818... - 0.181818...$ to give $99x = 18$	**M1**
$x = 18/99$ (oe)	**A1**
Simplified fully to $x = 2/11$ (cao)	**A1**

Answer: $x = 18/99 = 2/11$

Question 6

A valid counter-example value chosen, e.g. $n = 40$ (or $n = 41$)	**B1**
Expression correctly evaluated, e.g. $40^2 + 40 + 41 = 1681$	**M1**
1681 shown to be composite (e.g. $1681 = 41 \times 41$), so not prime, hence Kwame's statement is false (cao)	**A1**

Answer: $n = 40$ gives $1681 = 41 \times 41$, which is not prime, so Kwame is not correct.

Question 7

tests values of n in order or a counter example found	**M1**
$n = 11$ giving 121 with a factor shown	**A1**

Answer: $n = 11$: $121 = 11 \times 11$

Question 8

expands brackets correctly	**M1**
simplifies correctly	**M1**
$32n$ with a statement that a multiple of 32	**A1**

Answer: $32n$, which is a multiple of 32

Question 9

expands brackets correctly	**M1**
simplifies correctly	**M1**
$9(n + 4)$ with a statement that a multiple of 9	**A1**

Answer: $9(n + 4)$, which is a multiple of 9

Question 10

expands brackets correctly	**M1**
simplifies correctly	**M1**
$4n$ with a statement that a multiple of 4	**A1**

Answer: $4n$, which is a multiple of 4

Question 11

expands brackets correctly	**M1**
simplifies correctly	**M1**
$4n(n + 4)$ with a statement that a multiple of 4	**A1**

Answer: $4n(n + 4)$, which is a multiple of 4

Question 12

factorise $n^5 - n = n(n^4 - 1) = n(n^2 - 1)(n^2 + 1) = n(n - 1)(n + 1)(n^2 + 1)$	**M1**
state among n - 1, n, n + 1 one is even, so there is a factor 2	**M1**
state among n - 1, n, n + 1 one is multiple of 3, so there is a factor 3	**M1**
show divisibility by 5 by checking n mod 5 = 0, 1, 2, 3, 4 (or use Fermat): in each case $n^5 - n$ is a multiple of 5	**M1**
conclude $n^5 - n$ is divisible by 2, 3 and 5 and hence by 30 cao	**A1**

Answer: $n^5 - n = n(n - 1)(n + 1)(n^2 + 1)$ is divisible by 2 and 3 from the three consecutive factors, and by 5 by case or Fermat argument; therefore divisible by 30.

Question 13

$n^2 + n + 1$ evaluated correctly for at least three values of n in the range 1 to 5	**M1**
All five values correct: n=1 gives 3, n=2 gives 7, n=3 gives 13, n=4 gives 21, n=5 gives 31	**A1**
Each of the five values identified as odd	**B1**
Conclusion: since every integer from 1 to 5 has been checked (exhausted) and each gives an odd result, the statement is proved true for this range (cso)	**B1**

Answer: Values are 3, 7, 13, 21, 31, all odd, so $n^2 + n + 1$ is odd for every integer n from 1 to 5.

Question 14

$x^2 - 6x + 10$ written as $(x - 3)^2 - 9 + 10$ (oe)	**M1**
Simplified correctly to $(x - 3)^2 + 1$	**A1**
Statement that $(x - 3)^2 \geq 0$ for all real values of x	**M1**
Conclusion that $(x-3)^2 + 1 \geq 1 > 0$ for all real x, so $x^2 - 6x + 10 > 0$ for all real x (cso)	**A1**

Answer: $x^2 - 6x + 10 = (x-3)^2 + 1 \geq 1 > 0$ for all real x.

© Revision Library | revisionlibrary.co.uk