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HIGHER

8.3D Proof: Fluency and Exam Drill

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: ______________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 Prove that $n^2 - n$ is even for any integer n.

(Total for Question 1 is 2 marks)

2 Prove that $n^2 + n$ is even for any integer n.

(Total for Question 2 is 2 marks)

3 Show that the product of three consecutive integers $n(n + 1)(n + 2)$ is divisible by 3 for any integer n.

(Total for Question 3 is 2 marks)

4 Prove that $n^3 - n$ is divisible by 6 for any integer n.

(Total for Question 4 is 3 marks)

5 Prove that n and n^2 have the same parity. That is, show n is even if and only if n^2 is even, and n is odd if and only if n^2 is odd.

(Total for Question 5 is 3 marks)

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6 Show that the sum of three consecutive integers $n + (n + 1) + (n + 2)$ is divisible by 3 for any integer n.

(Total for Question 6 is 3 marks)

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7 Prove that $n^3 + 5n$ is divisible by 6 for any integer n.

(Total for Question 7 is 4 marks)

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8 Let n be an odd integer. Prove that $n^2 - 1$ is divisible by 8.

(Total for Question 8 is 4 marks)

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9 Show that the difference of consecutive square numbers $(n + 1)^2 - n^2$ is odd for any integer n.

(Total for Question 9 is 2 marks)

10 Stretch question. Prove that $n^5 - n$ is divisible by 30 for any integer n.

(Total for Question 10 is 5 marks)

Mark scheme · 8.3D Proof: Fluency and Exam Drill

Question 1

- **M1** factorise to $n(n - 1)$ or show product of two consecutive integers oe
- **A1** conclude $n(n - 1)$ is even, so $n^2 - n$ is even cao
- **Answer: $n(n - 1)$ is even, so $n^2 - n$ is even (for all integers n).**

Question 2

- **M1** factorise to $n(n + 1)$ or show product of two consecutive integers oe
- **A1** conclude $n(n + 1)$ is even, so $n^2 + n$ is even cao
- **Answer: $n(n + 1)$ is even, so $n^2 + n$ is even (for all integers n).**

Question 3

- **M1** note that among n, $n + 1$, $n + 2$ one is a multiple of 3 oe
- **A1** conclude $n(n + 1)(n + 2)$ is divisible by 3 cao
- **Answer: $n(n + 1)(n + 2)$ is divisible by 3.**

Question 4

- **M1** factorise to $n(n - 1)(n + 1)$ oe
- **M1** state that among three consecutive integers one is even and one is a multiple of 3 (so product divisible by 2 and by 3)
- **A1** conclude $n^3 - n$ is divisible by 6 cao
- **Answer: $n^3 - n = n(n - 1)(n + 1)$ is divisible by 6.**

Question 5

- **M1** show if $n = 2k$ then $n^2 = 4k^2$ which is even (even case)
- **M1** show if $n = 2k + 1$ then $n^2 = 4k^2 + 4k + 1$ which is odd (odd case)
- **A1** conclude n and n^2 have the same parity cao
- **Answer: If $n = 2k$ then n^2 is even; if $n = 2k + 1$ then n^2 is odd. So n and n^2 have the same parity.**

Question 6

- **M1** sum the three terms to get $3n + 3$
- **M1** factorise to $3(n + 1)$
- **A1** conclude the sum is divisible by 3 cao
- **Answer: $n + (n + 1) + (n + 2) = 3(n + 1)$, so divisible by 3.**

Question 7

- **M1** rewrite $n^3 + 5n$ as $(n^3 - n) + 6n$
- **M1** show $n^3 - n = n(n - 1)(n + 1)$, which is divisible by 6 (product of three consecutive integers)
- **M1** state $6n$ is divisible by 6
- **A1** conclude $n^3 + 5n$ is divisible by 6 cao
- **Answer: $n^3 + 5n = (n^3 - n) + 6n$, both parts divisible by 6, so $n^3 + 5n$ divisible by 6.**

Question 8

- **M1** substitute $n = 2k + 1$ and expand: $n^2 - 1 = (2k + 1)^2 - 1$
- **M1** simplify to $4k(k + 1)$
- **M1** state $k(k + 1)$ is even (product of consecutive integers) so has factor 2
- **A1** conclude $n^2 - 1 = 4k(k + 1)$ is divisible by 8 cao
- **Answer: If $n = 2k + 1$ then $n^2 - 1 = 4k(k + 1)$. Since $k(k + 1)$ is even, $n^2 - 1$ is divisible by 8.**

Question 9

- **M1** calculate $(n + 1)^2 - n^2 = 2n + 1$
- **A1** state $2n + 1$ is odd, so the difference is odd cao
- **Answer: $(n + 1)^2 - n^2 = 2n + 1$, which is odd for any integer n.**

Question 10

- **M1** factorise $n^5 - n = n(n^4 - 1) = n(n^2 - 1)(n^2 + 1) = n(n - 1)(n + 1)(n^2 + 1)$
- **M1** state among $n - 1, n, n + 1$ one is even, so there is a factor 2
- **M1** state among $n - 1, n, n + 1$ one is multiple of 3, so there is a factor 3
- **M1** show divisibility by 5 by checking $n \bmod 5 = 0, 1, 2, 3, 4$ (or use Fermat): in each case $n^5 - n$ is a multiple of 5
- **A1** conclude $n^5 - n$ is divisible by 2, 3 and 5 and hence by 30 cao
- **Answer: $n^5 - n = n(n - 1)(n + 1)(n^2 + 1)$ is divisible by 2 and 3 from the three consecutive factors, and by 5 by case or Fermat argument; therefore divisible by 30.**