HIGHER

8.4 Completing the Square: 1-hour Pack

EDEXCEL 1MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

Completing the Square: Method Reminder

Revision Library original resource, written for Edexcel 1MA1 topic 8.4.

To complete the square for x^2 + bx + c, halve the coefficient of x to get b/2, then write (x + b/2)^2 - (b/2)^2 + c. For ax^2 + bx + c with a not equal to 1, first factorise a out of the x^2 and x terms before completing the square inside the bracket, then multiply back through. Once in the form a(x+p)^2 + q, you can read off the turning point of the graph directly: minimum (or maximum) at (-p, q). To solve ax^2 + bx + c = 0, isolate the squared bracket and take a square root of both sides, remembering the +/- sign.

1 Write $x^2 + 9x$ in the form $(x + p)^2 + q$, where p and q may be fractions.

(Total for Question 1 is 3 marks)

2 Write $x^2 - 7x + 1$ in the form $(x + p)^2 + q$.

(Total for Question 2 is 3 marks)

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3 Write $5x^2 + 20x + 3$ in the form $a(x + p)^2 + q$, stating the values of a, p and q.

(Total for Question 3 is 4 marks)

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4 The height, h metres, of a ball t seconds after being thrown is modelled by $h = -5t^2 + 20t + 1$.

(a) Write $-5t^2 + 20t + 1$ in the form $-5(t - p)^2 + q$. **(4)**

(b) Hence, or otherwise, find the maximum height of the ball and the time at which it occurs. **(2)**

(Total for Question 4 is 6 marks)

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5 Consider the equation $4x^2 + 16x + 15 = 0$.

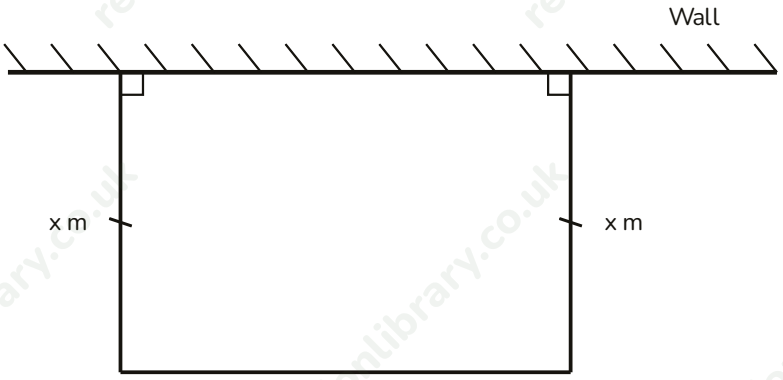
(a) Show that $4x^2 + 16x + 15$ can be written as $(2x + 4)^2 - 1$. **(3)**

(b) Hence solve $4x^2 + 16x + 15 = 0$, giving your answers as exact fractions. **(2)**

(Total for Question 5 is 5 marks)

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6 A gardener has 24 metres of fencing. She uses it to enclose a rectangular vegetable patch on three sides, using an existing straight wall as the fourth side. The width of the patch, perpendicular to the wall, is x metres.



Schematic, not to scale

(a) Show that the area, A square metres, of the patch is given by $A = 24x - 2x^2$. **(2)**

(b) Express A in the form $q - 2(x - p)^2$, by completing the square. **(3)**

(c) Hence find the maximum possible area of the patch and the width, x, for which this occurs. **(2)**

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(Total for Question 6 is 7 marks)

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7 The function f is defined by $f(x) = x^2 - 3x + 7$.

(a) Express f(x) in the form $(x - a)^2 + b$, where a and b are exact values. **(3)**

(b) Hence state the minimum value of f(x). **(1)**

(Total for Question 7 is 4 marks)

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8 The curve with equation $y = x^2 + px + q$ has a minimum point at (5, -3). Find the values of p and q.

(Total for Question 8 is 4 marks)

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9 Prove algebraically that $x^2 - 6x + 11 \geq 2$ for all real values of x.

(Total for Question 9 is 3 marks)

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10 Solve $x^2 + 5x - 3 = 0$ by completing the square. Give your answers in the form $(a \pm \sqrt{b}) / c$, where a, b and c are integers.

(Total for Question 10 is 4 marks)

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11 By completing the square, find the minimum value of $x^2 + 4x + 17$ and the value of x for which it occurs.

(Total for Question 11 is 3 marks)

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Mark scheme · 8.4 Completing the Square: 1-hour Pack

A correct answer earns full marks unless the question asks for working, a method, proof or explanation.

Question 1

attempt to halve 9 to get 9/2 oe	**M1**
$(x + 9/2)^2$ seen	**M1**
$(x + 9/2)^2$ - 81/4 cao (accept -20.25)	**A1**

Answer: $(x + 9/2)^2$ - 81/4

Question 2

$(x - 7/2)^2$ seen	**M1**
attempt to combine -49/4 with +1	**M1**
$(x - 7/2)^2$ - 45/4 cao (accept -11.25)	**A1**

Answer: $(x - 7/2)^2$ - 45/4

Question 3

$5(x^2 + 4x) + 3$ seen, factor of 5 taken out correctly	**M1**
$(x + 2)^2$ seen within the bracket	**M1**
$5(x + 2)^2$ - 20 + 3 correctly simplified (dep. on both M marks)	**A1**
a = 5, p = 2, q = -17 all correct, fully simplified $5(x + 2)^2$ - 17 cao	**A1**

Answer: $5(x + 2)^2$ - 17 (a = 5, p = 2, q = -17)

Question 4

(a) $-5(t^2 - 4t) + 1$ oe, factor of -5 taken out correctly	**M1**
(a) $-5[(t - 2)^2 - 4] + 1$ oe	**M1**
(a) correct expansion: $-5(t-2)^2 + 20 + 1$	**M1**
(a) $-5(t - 2)^2 + 21$ cao	**A1**

(a) Answer: $-5(t - 2)^2 + 21$

(b) ft maximum height = 21 metres	**B1**
(b) ft occurs when t = 2 seconds	**B1**

(b) Answer: Maximum height 21 m, at t = 2 seconds

Question 5

(a) $(2x + 4)^2 = 4x^2 + 16x + 16$ obtained	**M1**
(a) 16 - 1 = 15 identified	**M1**
(a) cso: $(2x+4)^2 - 1 = 4x^2 + 16x + 15$, answer printed	**A1**

(a) Answer: $(2x + 4)^2$ - 1, as shown

(b) ft $(2x + 4)^2 = 1$ leading to $2x + 4 = \pm 1$	**M1**
(b) x = -3/2 and x = -5/2 oe, both exact	**A1**

(b) Answer: x = -3/2 or x = -5/2

Question 6

(a) length parallel to the wall = 24 - 2x oe	**M1**
(a) cso: $A = x(24 - 2x) = 24x - 2x^2$, answer printed	**A1**

(a) Answer: $A = 24x - 2x^2$, as shown

(b) $-2(x^2 - 12x)$ oe, factor of -2 taken out correctly	**M1**
(b) $-2[(x - 6)^2 - 36]$ oe	**M1**
(b) $72 - 2(x - 6)^2$ cao	**A1**

(b) Answer: $72 - 2(x - 6)^2$

(c) ft maximum area = 72 square metres	**B1**
(c) ft occurs when x = 6	**B1**

(c) Answer: Maximum area 72 m^2, when x = 6 m

Question 7

(a) $(x - 1.5)^2$ oe seen (allow x - 3/2)	**M1**
(a) correct subtraction of $(1.5)^2 = 2.25$ from 7	**M1**
(a) $(x - 1.5)^2 + 4.75$ oe (accept $(x - 3/2)^2 + 19/4$)	**A1**

(a) Answer: $(x - 1.5)^2 + 4.75$

(b) ft 4.75 oe (accept 19/4)	**B1**

(b) Answer: 4.75

Question 8

recognise $y = (x - 5)^2 - 3$, using the given minimum point to write the completed-square form	**M1**
expand $(x - 5)^2$ to get $x^2 - 10x + 25$	**M1**
p = -10 cao	**A1**
q = 22 cao	**A1**

Answer: p = -10, q = 22

Question 9

completes the square: $x^2 - 6x + 11 = (x - 3)^2 - 9 + 11$	**M1**
$(x - 3)^2 + 2$ obtained	**A1**
dep on M1: since $(x - 3)^2 \geq 0$ for all real x, $(x - 3)^2 + 2 \geq 2$, so $x^2 - 6x + 11 \geq 2$ for all real x, cso	**B1**

Answer: $x^2 - 6x + 11 = (x-3)^2 + 2 \geq 2$, since $(x-3)^2 \geq 0$ for all real x

Question 10

$(x + 2.5)^2 - 6.25 - 3 = 0$ oe, correct completed square set to zero	**M1**
$(x + 2.5)^2 = 9.25$ oe (accept 37/4)	**M1**
$x + 2.5 = +\text{-}\sqrt{37}/2$ oe, correct rearrangement	**M1**
$x = (-5 + \sqrt{37}) / 2$ and $x = (-5 - \sqrt{37}) / 2$ oe, both exact	**A1**

Answer: $x = (-5 + \sqrt{37}) / 2$ or $x = (-5 - \sqrt{37}) / 2$

Question 11

completes the square	**M1**
minimum value 13	**A1**
x = -2	**A1**

Answer: minimum value = 13 when x = -2

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