HIGHER

# 8.4 Completing the Square: Test Pack

EDEXCEL 1MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

**Answer ALL questions. Show all your working.**

## Completing the Square: Method Reminder

*Revision Library original resource, written for Edexcel 1MA1 topic 8.4.*

To complete the square for x^2 + bx + c, halve the coefficient of x to get b/2, then write (x + b/2)^2 - (b/2)^2 + c. For ax^2 + bx + c with a not equal to 1, first factorise a out of the x^2 and x terms before completing the square inside the bracket, then multiply back through. Once in the form a(x+p)^2 + q, you can read off the turning point of the graph directly: minimum (or maximum) at (-p, q). To solve ax^2 + bx + c = 0, isolate the squared bracket and take a square root of both sides, remembering the +/- sign.

**1** Write $x^2 + 16x$ in the form $(x + p)^2 + q$.

**(Total for Question 1 is 2 marks)**

**2** By completing the square, find the minimum value of $f(x) = x^2 + 4x + 9$, and state the value of x at which this minimum occurs.

**(Total for Question 2 is 3 marks)**


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**3** Solve $x^2 + 14x + 9 = 0$ by completing the square. Give your answers in surd form, simplified where possible.

**(Total for Question 3 is 4 marks)**

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**4** The expression $x^2 + 2x - 8$ is to be written in completed-square form.

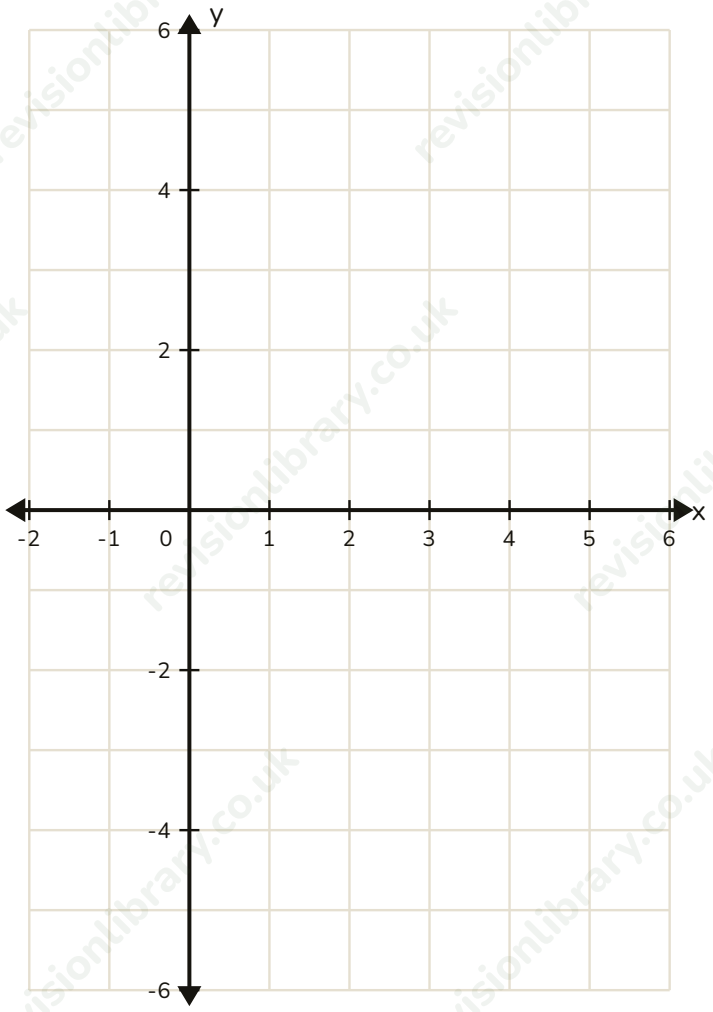
**(a)** Write $x^2 + 2x - 8$ in the form $(x + p)^2 + q$. **(2)**

**(b)** Hence solve $x^2 + 2x - 8 = 0$, showing your method clearly. **(3)**

**(Total for Question 4 is 5 marks)**


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**5** The curve C has equation $y = 2x^2 - 8x + 3$.




**(a)** Express $2x^2 - 8x + 3$ in the form $2(x - a)^2 + b$. **(3)**

**(b)** Write down the coordinates of the minimum point of C. **(1)**

.....

**(c)** On the grid provided, sketch the graph of C, showing clearly the coordinates of the minimum point and the y-intercept. **(2)**

**(Total for Question 5 is 6 marks)**


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**6** A market trader models the weekly profit, L pounds, from selling padlocks at a price of x pounds each, using $L = -3x^2 + 30x - 39$.

**(a)** Show that L can be written as $-3(x - 5)^2 + 36$. **(2)**

**(b)** Hence find the price, x, that maximises the weekly profit, and state the maximum profit. **(2)**

.....

**(Total for Question 6 is 4 marks)**

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**7** The curve $y = 4x^2 - 24x + c$ has a minimum value of 11. Find the value of c, and state the coordinates of the minimum point.

**(Total for Question 7 is 6 marks)**

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**8** Prove that the equation $x^2 + 6x + 10 = 0$ has no real roots.

**(Total for Question 8 is 3 marks)**


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**9** By completing the square, find the minimum value of $x^2 - 6x + 12$ and the value of x for which it occurs.

**(Total for Question 9 is 3 marks)**

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**10** Solve $x^2 + 6x + 3 = 0$ by completing the square. Give your solutions in the form $p + \sqrt{q}$ and $p - \sqrt{q}$.

**(Total for Question 10 is 4 marks)**

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**11** Prove that the equation $x^2 + 2kx + k^2 + 3 = 0$, where k is a constant, has no real solutions for any value of k.

**(Total for Question 11 is 4 marks)**


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# Mark scheme · 8.4 Completing the Square: Test Pack

A correct answer earns full marks unless the question asks for working, a method, proof or explanation.

## Question 1

| | |
|---|---|
| $(x + 8)^2$ seen, or correct method to halve the coefficient of x | **M1** |
| $(x + 8)^2 - 64$ cao | **A1** |

**Answer: $(x + 8)^2 - 64$**

## Question 2

| | |
|---|---|
| $(x + 2)^2$ seen, complete the square attempted | **M1** |
| $(x + 2)^2 + 5$ correct, hence minimum value = 5 | **A1** |
| ft x = -2 (follow through from their p value) | **B1** |

**Answer: minimum value = 5, at x = -2**

## Question 3

| | |
|---|---|
| $(x + 7)^2 - 49 + 9 = 0$ oe, correct completed square set to zero | **M1** |
| $(x + 7)^2 = 40$ (dep. on previous M1) | **M1** |
| $x + 7 = \pm \sqrt{40}$, condone unsimplified surd | **M1** |
| $x = -7 + 2\sqrt{10}$ or $x = -7 - 2\sqrt{10}$ oe, both fully simplified, cao | **A1** |

**Answer: $x = -7 + 2\sqrt{10}$ or $x = -7 - 2\sqrt{10}$**

## Question 4

| | |
|---|---|
| (a) $(x + 1)^2$ seen | **M1** |
| (a) $(x + 1)^2 - 9$ cao | **A1** |

**(a) Answer: $(x + 1)^2 - 9$**

| | |
|---|---|
| (b) ft $(x + 1)^2 = 9$, using the answer to part (a) | **M1** |
| (b) $x + 1 = \pm 3$ | **M1** |
| (b) x = 2 and x = -4, both correct, cao | **A1** |

**(b) Answer: x = 2 or x = -4**

## Question 5

| | |
|---|---|
| (a) $2(x^2 - 4x) + 3$ oe | **M1** |
| (a) $2[(x - 2)^2 - 4] + 3$ oe | **M1** |
| (a) $2(x - 2)^2 - 5$ cao | **A1** |

**(a) Answer: $2(x - 2)^2 - 5$**

| | |
|---|---|
| (b) ft (2, -5) cao | **B1** |

**(b) Answer: (2, -5)**

| | |
|---|---|
| (c) ft correct u-shape with minimum point plotted at (2, -5) | **B1** |
| (c) y-intercept plotted at (0, 3) | **B1** |

**(c) Answer: U-shaped curve, minimum (2, -5), y-intercept (0, 3)**

## Question 6

| | |
|---|---|
| (a) $-3(x^2 - 10x) - 39$ seen, factor of -3 taken out correctly | **M1** |
| (a) $-3(x - 5)^2 + 36$ correctly derived, showing $-3[(x-5)^2 - 25] - 39 = -3(x-5)^2 + 75 - 39$ | **A1** |

**(a) Answer: $L = -3(x - 5)^2 + 36$**

| | |
|---|---|
| (b) ft x = 5 (pounds), using $-3(x-5)^2 \leq 0$ for all real x so L is greatest when $(x-5)^2 = 0$ | **B1** |
| (b) maximum profit = 36 (pounds) | **A1** |

**(b) Answer: x = £5; maximum profit = £36**

## Question 7

| | |
|---|---|
| $4(x^2 - 6x) + c$ seen, factor of 4 taken out correctly | **M1** |
| $(x - 3)^2$ seen within the bracket | **M1** |
| $4(x - 3)^2 - 36 + c$ correctly simplified expression for y | **A1** |
| dep: set $-36 + c = 11$, using their simplified constant term | **M1** |
| c = 47 cao | **A1** |
| ft minimum point = (3, 11) | **B1** |

**Answer: c = 47; minimum point (3, 11)**

## Question 8

| | |
|---|---|
| completes the square: $x^2 + 6x + 10 = (x + 3)^2 - 9 + 10$ | **M1** |
| $(x + 3)^2 + 1 = 0$ leading to $(x + 3)^2 = -1$ | **A1** |
| dep: since $(x + 3)^2 \geq 0$ for all real x, it can never equal -1, so the equation has no real roots, cso | **B1** |

**Answer: $(x+3)^2 = -1$ has no real solution, since a square cannot be negative; hence no real roots**

## Question 9

| | |
|---|---|
| completes the square | **M1** |
| minimum value 3 | **A1** |
| x = 3 | **A1** |

**Answer: minimum value = 3 when x = 3**

## Question 10

| | |
|---|---|
| $(x + 3)^2$ seen | **M1** |
| $(x + 3)^2 = 6$ | **M1** |
| $x + 3 = \pm \sqrt{6}$ | **M1** |
| $x = -3 + \sqrt{6}$ or $x = -3 - \sqrt{6}$ | **A1** |

**Answer: $x = -3 + \sqrt{6}$ or $x = -3 - \sqrt{6}$**

## Question 11

| | |
|---|---|
| recognise $x^2 + 2kx + k^2 = (x + k)^2$, a perfect square | **M1** |
| equation rewritten as $(x + k)^2 + 3 = 0$ oe | **A1** |
| dep: valid reasoning that $(x + k)^2 \geq 0$ for all real x (and all real k), so $(x + k)^2 + 3 \geq 3$ | **M1** |
| cso: hence $(x + k)^2 + 3$ cannot equal 0, since it is always $\geq 3 > 0$, so the equation has no real solutions for any value of k | **A1** |

**Answer: $(x + k)^2 + 3 = 0$ has no real solutions, since $(x + k)^2 + 3 \geq 3 > 0$ for all real x and k**


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