HIGHER

8.5 The Nth Term of a Quadratic Sequence: Test Pack

Total Marks

EDEXCEL 1MA1 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Write down the second difference for the sequence 4, 9, 16, 25, 36.

(Total for Question 1 is 1 mark)

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2 Work out an expression for the nth term of the sequence 1, 6, 15, 28.

(Total for Question 2 is 2 marks)

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3 The number of diagonals, D, in a polygon with n sides is given by the formula $D = n(n-3)/2$.

(a) Show that this formula can be written as $D = 0.5n^2 - 1.5n$. **(2)**

(b) Work out the number of diagonals in a polygon with 12 sides. **(2)**

(c) A polygon has 90 diagonals. Form and solve an equation to find the number of sides of the polygon. **(4)**

(Total for Question 3 is 8 marks)

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4 A bakery in Leeds models its monthly profit, P, in thousands of pounds, in month n by $P(n) = -2n^2 + 40n - 150$, for $1 \leq n \leq 15$, where n = 1 represents January 2020 and each increase of 1 in n represents one further year (so n=1 is 2020, n=2 is 2021, and so on).

(a) Work out the profit predicted for 2023. **(2)**

.....

(b) Work out the maximum profit the model predicts, and state the year in which it occurs. **(3)**

.....

(c) The bakery will close if the profit is negative for two or more consecutive years. Determine whether the model predicts the bakery will close within the given range of n. **(4)**

(Total for Question 4 is 9 marks)

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5 The nth term of a sequence is $n^2 + 5n + 4$. Is 180 a term of this sequence? You must show your working.

(Total for Question 5 is 3 marks)

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6 The first four terms of a quadratic sequence are 2, 1, -4, -13. Find the nth term of the sequence and hence work out the 14th term.

(Total for Question 6 is 5 marks)

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7 A quadratic sequence has nth term $n^2 + 4$. Zara says, 'The number 68 is a term in this sequence.' Determine whether Zara is correct.

(Total for Question 7 is 3 marks)

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8 The nth term of a sequence is $4n^2 - 7n + 2$. Find the smallest positive integer n for which the term is greater than 200.

(Total for Question 8 is 3 marks)

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9 The number of squares, S, needed to build Shape n in a growing pattern is given by $S = n^2 + n + 2$.

(a) Deema builds Shape 6. Work out how many squares she needs. **(2)**

.....

(b) Deema has 100 squares. Work out the largest shape number she can build without running out of squares. **(3)**

.....

(Total for Question 9 is 5 marks)

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10 Sequence P has nth term $2n^2 - 5n + 3$. Sequence Q has nth term $n^2 + 4n - 17$. Find the value of n for which the nth term of Sequence P is 20 more than the nth term of Sequence Q.

(Total for Question 10 is 4 marks)

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11 A quadratic sequence has nth term $n^2 + n + 8$. Priya says, 'Every term in this sequence is even.' Prove that Priya is correct.

(Total for Question 11 is 3 marks)

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Mark scheme · 8.5 The Nth Term of a Quadratic Sequence: Test Pack

Question 1

second difference = 2 cao **B1**

Answer: 2

Question 2

second differences 5, 9, 13 so second difference = 4 and a = 2 (2a = 4) oe **M1**

nth term = $2n^2$ - n cao **A1**

Answer: $2n^2$ - n

Question 3

(a) expanding n(n-3) = n^2 - 3n and dividing by 2 **M1**

(a) cso: $0.5n^2$ - 1.5n **A1**

(a) Answer: $0.5n^2$ - 1.5n

(b) substituting n=12 into n(n-3)/2 **M1**

(b) 54 cao **A1**

(b) Answer: 54

(c) forming the equation n(n-3)/2 = 90, i.e. n^2 - 3n - 180 = 0 **M1**

(c) solving using factorisation or the quadratic formula **M1**

(c) n = 15 cao **A1**

(c) rejecting n = -12 since the number of sides must be a positive whole number **A1**

(c) Answer: 15 sides

Question 4

(a) substituting n=4 into $-2n^2$+40n-150 **M1**

(a) -22 cao, i.e. a loss of £22,000 **A1**

(a) Answer: -22 (a loss of £22,000)

(b) finding n=10 using n=-b/(2a) or completing the square **M1**

(b) substituting n=10 to find P(10)=50 **M1**

(b) correct link of n=10 to the year 2029 **A1**

(b) Answer: Maximum profit £50,000 in 2029

(c) setting up $-2n^2$+40n-150 < 0 and simplifying to n^2-20n+75 > 0 **M1**

(c) solving n^2-20n+75=0 to find the critical values n=5 and n=15 **M1**

(c) correctly identifying that profit is negative for n=1,2,3,4 (i.e. n<5) within the given range **M1**

(c) correct conclusion: yes, the model predicts closure, since 2020-2023 (4 consecutive years) all have negative profit **A1**

(c) Answer: Yes, the bakery is predicted to close

Question 5

n^2 + 5n + 4 = 180 **M1**

tests values of n or solves the quadratic **M1**

Yes, term number 11 with supporting working **A1**

Answer: Yes, term number 11

Question 6

first differences -1, -5, -9 **M1**

second difference -4, so a = -2 **M1**

subtracts the quadratic part to find the linear part **M1**

$-2n^2$ + 5n - 1 **A1**

-323 **A1**

Answer: $-2n^2$ + 5n - 1; term 14 = -323

Question 7

forming the equation n^2 + 4 = 68, i.e. n^2 = 64 **M1**

solving to get n = 8 (or n = ±8) **M1**

correct conclusion: since n=8 is a positive integer, Zara is correct (cao) **A1**

Answer: Zara is correct (n = 8)

Question 8

set inequality $4n^2$ - 7n + 2 > 200 and rearrange to $4n^2$ - 7n - 198 > 0 oe **M1**

find positive root of $4n^2$ - 7n - 198 = 0 approximately (n approx 7.97) oe **M1**

smallest integer n = 8 cao **A1**

Answer: 8

Question 9

(a) substituting n=6 into n^2+n+2 **M1**

(a) 44 cao **A1**

(a) Answer: 44

(b) forming the equation or inequality n^2 + n + 2 = 100 (or ≤ 100) **M1**

(b) solving using the quadratic formula to obtain n = 9.41 (2 dp) or equivalent **M1**

(b) n = 9, with justification that Shape 10 needs 112 squares which is too many **A1**

(b) Answer: Shape 9

Question 10

setting up $2n^2$-5n+3 = (n^2+4n-17) + 20 **M1**

rearranging to n^2 - 9n = 0 **M1**

factorising n(n-9) = 0 **M1**

n = 9 cao (rejecting n=0 as not a valid term number) **A1**

Answer: n = 9

Question 11

factorising n^2 + n = n(n+1) **M1**

stating that n and n+1 are consecutive integers, so one of them must be even, meaning n(n+1) is always even **M1**

cso: concluding that n(n+1) + 8 is the sum of two even numbers, so it is always even, proving Priya is correct **A1**

Answer: Priya is correct; every term is even

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