HIGHER

8.9 Perpendicular Lines and the equation of a tangent: 2-hour Pack

Total Marks

EDEXCEL 1MA1 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The point P(-11, 7) lies on the circle $x^2 + y^2 = 170$. Find an equation of the tangent to the circle at P.

(Total for Question 1 is 4 marks)

2 A straight line has equation $-3x - 5y = 1$. Find an equation of the straight line that is perpendicular to this line and passes through the point (6, 4).

(Total for Question 2 is 4 marks)

3 A is the point (-2, 8) and B is the point (6, 0). Find an equation of the perpendicular bisector of AB.

(Total for Question 3 is 5 marks)

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4 A straight line L has equation $y = -(2/3)x + 3$. Find an equation of the straight line that is perpendicular to L and passes through the point (-2, 0).

(Total for Question 4 is 3 marks)

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5 A straight line has equation $-4x + y = 5$. Find an equation of the straight line that is perpendicular to this line and passes through the point (-2, -1).

(Total for Question 5 is 4 marks)

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6 The circle with equation $(x + 4)^2 + (y + 2)^2 = 145$ passes through the point P(-3, 10). Find an equation of the tangent to the circle at P.

(Total for Question 6 is 5 marks)

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7 A straight line L has equation $y = -x - 3$. Find an equation of the straight line that is perpendicular to L and passes through the point (-1, -6).

(Total for Question 7 is 3 marks)

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8 A straight line has equation $x + 3y = -12$. Find an equation of the straight line that is perpendicular to this line and passes through the point (3, 6).

(Total for Question 8 is 4 marks)

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9 A straight line L has equation $y = (5/3)x$. Find an equation of the straight line that is perpendicular to L and passes through the point (0, -1).

(Total for Question 9 is 3 marks)

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10 A is the point (6, -6) and B is the point (8, 4). Find an equation of the perpendicular bisector of AB.

(Total for Question 10 is 5 marks)

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11 A straight line has equation $3x - 3y = -10$. Find an equation of the straight line that is perpendicular to this line and passes through the point (-6, -5).

(Total for Question 11 is 4 marks)

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12 The point P(-17, 4) lies on the circle $x^2 + y^2 = 305$. Find an equation of the tangent to the circle at P.

(Total for Question 12 is 4 marks)

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13 A circle with centre O, the origin, has equation $x^2 + y^2 = 50$
The point P(-5, 5) lies on the circle.
Find the equation of the tangent to the circle at P. Give your answer in the form $y = mx + c$

(Total for Question 13 is 4 marks)

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14 A circle with centre O, the origin, has equation $x^2 + y^2 = 34$
The point (5, 3) lies on the circle.
Find the equation of the tangent to the circle at this point. Give your answer in the form $ax + by = c$, where a, b and c are integers.

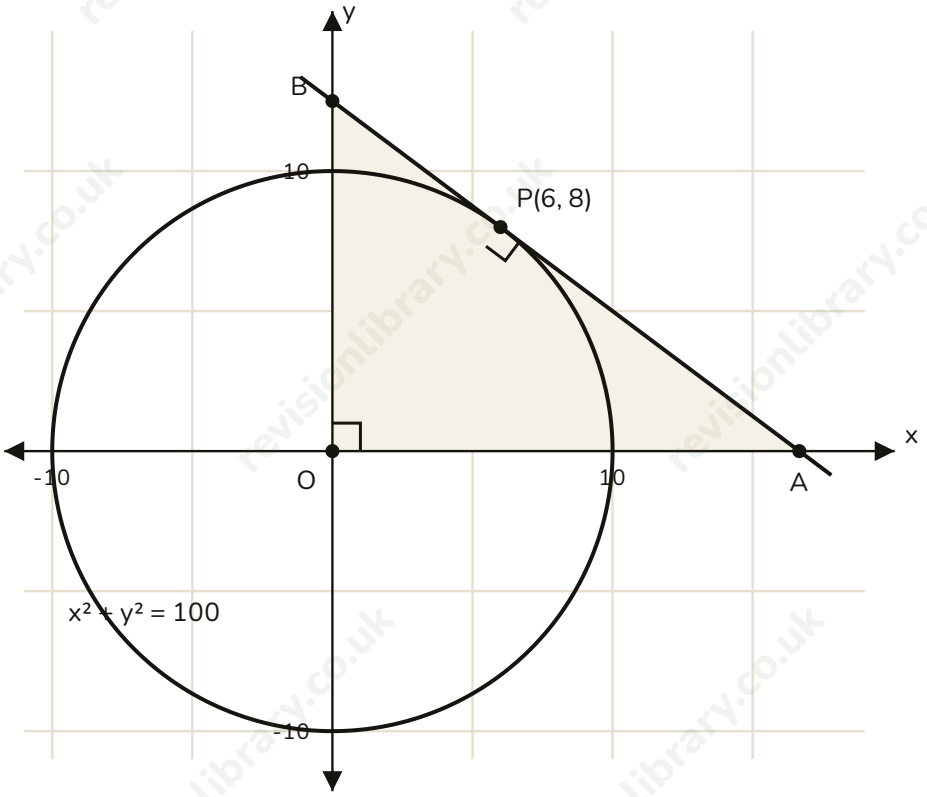
(Total for Question 14 is 4 marks)

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15 A circle with centre O, the origin, has equation $x^2 + y^2 = 100$

The tangent to the circle at the point P(6, 8) crosses the x-axis at point A and crosses the y-axis at point B.

Diagram NOT accurately drawn



(a) Find the equation of the tangent to the circle at P. **(3)**

.....

(b) Find the coordinates of A. **(1)**

.....

(c) Find the coordinates of B. **(1)**

.....

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(d) Calculate the area of triangle OAB, where O is the origin. Give your answer as an exact fraction or correct to 3 significant figures. **(3)**

.....

(Total for Question 15 is 8 marks)

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16 A circle with centre O, the origin, has equation $x^2 + y^2 = 25$
P is the point (-3, 4).

(a) Find the equation of the tangent to the circle at P. **(4)**

.....

(b) This tangent intersects the line with equation $y = 2x + 1$ at a single point. Find the coordinates of this point. **(3)**

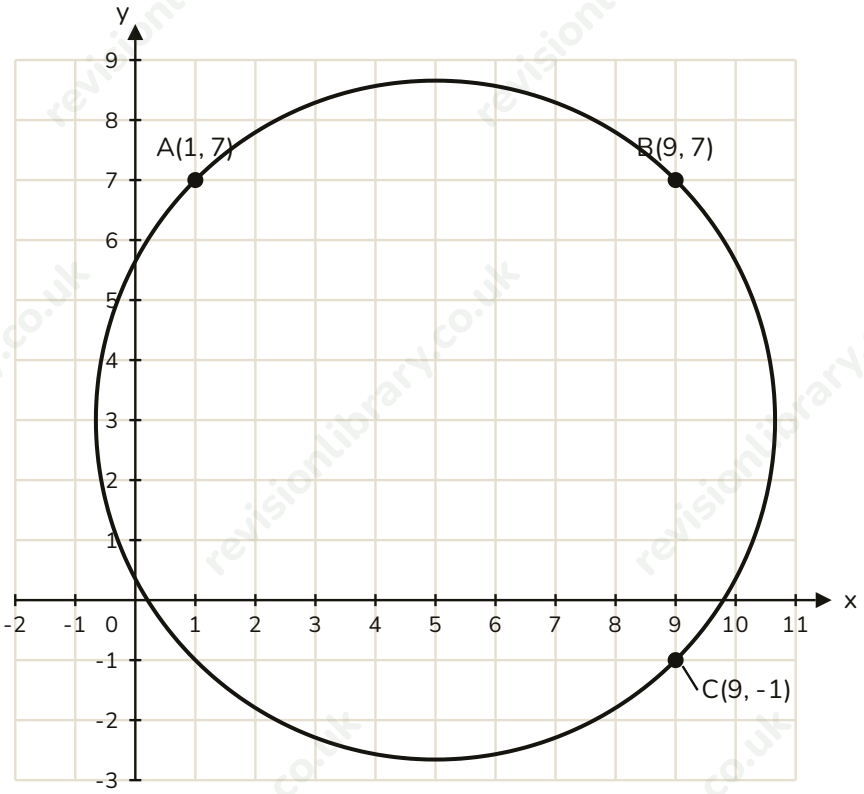
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(Total for Question 16 is 7 marks)

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17 A circle passes through the points A(1, 7), B(9, 7) and C(9, -1).

Diagram NOT accurately drawn



(a) Find the equation of the perpendicular bisector of AB. **(2)**

.....

(b) Find the equation of the perpendicular bisector of BC. **(2)**

.....

(c) Hence find the coordinates of the centre of the circle. **(2)**

.....

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(d) Find the radius of the circle, and hence write down the equation of the circle. **(3)**

.....

(Total for Question 17 is 9 marks)

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Mark scheme · 8.9 Perpendicular Lines and the equation of a tangent: 2-hour Pack

Question 1

gradient of OP = -7/11	**M1**
gradient of the tangent = -1/(-7/11) = 11/7	**M1**
substitutes (-11, 7) into y = mx + c	**M1**
y = (11/7)x + 170/7 oe	**A1**

Answer: y = (11/7)x + 170/7

Question 2

rearranges to find the gradient -3/5	**M1**
perpendicular gradient 5/3	**M1**
substitutes the point	**M1**
y = (5/3)x - 6 oe	**A1**

Answer: y = (5/3)x - 6

Question 3

midpoint (2, 4)	**M1**
gradient of AB = -1	**M1**
perpendicular gradient 1	**M1**
substitutes the midpoint into y = mx + c	**M1**
y = x + 2 oe	**A1**

Answer: y = x + 2

Question 4

gradient of the perpendicular = -1/(-2/3) = 3/2	**M1**
substitutes (-2, 0) to find the intercept	**M1**
y = (3/2)x + 3 oe	**A1**

Answer: y = (3/2)x + 3

Question 5

rearranges to find the gradient 4	**M1**
perpendicular gradient -1/4	**M1**
substitutes the point	**M1**
y = -(1/4)x - 3/2 oe	**A1**

Answer: y = -(1/4)x - 3/2

Question 6

centre (-4, -2) identified	**M1**
gradient of the radius = 12	**M1**
gradient of the tangent = -1/12	**M1**
substitutes (-3, 10)	**M1**
y = -(1/12)x + 39/4 oe	**A1**

Answer: y = -(1/12)x + 39/4

Question 7

gradient of the perpendicular = -1/(-1) = 1	**M1**
substitutes (-1, -6) to find the intercept	**M1**
y = x - 5 oe	**A1**

Answer: y = x - 5

Question 8

rearranges to find the gradient -1/3	**M1**
perpendicular gradient 3	**M1**
substitutes the point	**M1**
y = 3x - 3 oe	**A1**

Answer: y = 3x - 3

Question 9

gradient of the perpendicular = -1/(5/3) = -3/5	**M1**
substitutes (0, -1) to find the intercept	**M1**
y = -(3/5)x - 1 oe	**A1**

Answer: y = -(3/5)x - 1

Question 10

midpoint (7, -1)	**M1**
gradient of AB = 5	**M1**
perpendicular gradient -1/5	**M1**
substitutes the midpoint into y = mx + c	**M1**
y = -(1/5)x + 2/5 oe	**A1**

Answer: y = -(1/5)x + 2/5

Question 11

rearranges to find the gradient 1	**M1**
perpendicular gradient -1	**M1**
substitutes the point	**M1**
y = -x - 11 oe	**A1**

Answer: y = -x - 11

Question 12

gradient of OP = -4/17	**M1**
gradient of the tangent = -1/(-4/17) = 17/4	**M1**
substitutes (-17, 4) into y = mx + c	**M1**
y = (17/4)x + 305/4 oe	**A1**

Answer: y = (17/4)x + 305/4

Question 13

gradient of OP = 5/(-5) = -1 (oe)	**M1**
tangent gradient = 1 (negative reciprocal of -1)	**M1**
correct method using point P, e.g. y - 5 = 1(x + 5)	**M1**
y = x + 10 (cao)	**A1**

Answer: y = x + 10

Question 14

gradient of radius = 3/5	**M1**
tangent gradient = -5/3 (negative reciprocal of 3/5)	**M1**
correct method using the point (5, 3), e.g. y - 3 = -(5/3)(x - 5)	**M1**
5x + 3y = 34 (oe, integer coefficients) (cao)	**A1**

Answer: 5x + 3y = 34

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Question 15

(a) gradient of OP = 8/6 = 4/3, so tangent gradient = -3/4 **M1**

(a) correct method using point P, e.g. y - 8 = -(3/4)(x - 6) **M1**

(a) y = -(3/4)x + 12.5 (oe, e.g. 3x + 4y = 50) (cao) **A1**

(a) Answer: y = -(3/4)x + 12.5

(b) (50/3, 0) (oe awrt (16.7, 0)), ft their part (a) **A1**

(b) Answer: A = (50/3, 0)

(c) (0, 12.5) (oe (0, 25/2)), ft their part (a) **A1**

(c) Answer: B = (0, 12.5)

(d) recognises that OA and OB are perpendicular (along the axes), so area = (1/2) x OA x OB **M1**

(d) substitutes their values, e.g. (1/2) x (50/3) x 12.5 **M1**

(d) 625/6 (oe awrt 104) square units, ft their parts (b) and (c) **A1**

(d) Answer: Area = 625/6 square units (awrt 104 square units)

Question 16

(a) gradient of OP = 4/(-3) = -4/3, so tangent gradient = 3/4 **M1**

(a) correct method using point P, e.g. y - 4 = (3/4)(x + 3) **M1**

(a) correct rearrangement towards integer form **M1**

(a) -3x + 4y = 25 (oe, e.g. y = (3/4)x + 25/4) (cao) **A1**

(a) Answer: -3x + 4y = 25 (oe y = (3/4)x + 25/4)

(b) sets their tangent equation equal to 2x + 1, ft their part (a) **P1**

(b) correct algebraic method to solve for x, e.g. (3/4)x + 25/4 = 2x + 1 leading to 3x + 25 = 8x + 4 **M1**

(b) (21/5, 47/5) (oe (4.2, 9.4)), both coordinates correct (cao) **A1**

(b) Answer: (21/5, 47/5) = (4.2, 9.4)

Question 17

(a) finds the midpoint of AB as (5, 7) and recognises AB is horizontal, so its perpendicular bisector is vertical **M1**

(a) x = 5 (cao) **A1**

(a) Answer: x = 5

(b) finds the midpoint of BC as (9, 3) and recognises BC is vertical, so its perpendicular bisector is horizontal **M1**

(b) y = 3 (cao) **A1**

(b) Answer: y = 3

(c) recognises the centre lies on both perpendicular bisectors, so solves x = 5 and y = 3 simultaneously, ft their parts (a) and (b) **M1**

(c) (5, 3) (cao) **A1**

(c) Answer: (5, 3)

(d) uses the distance formula from their centre to A(1, 7) (or B or C), e.g. $\sqrt{(5-1)^2 + (3-7)^2}$ **M1**

(d) radius = $\sqrt{32}$ (oe $4\sqrt{2}$), ft their part (c) **A1**

(d) $(x - 5)^2 + (y - 3)^2 = 32$ (cao, ft) **A1**

(d) Answer: radius = $\sqrt{32} = 4\sqrt{2}$; circle equation: $(x - 5)^2 + (y - 3)^2 = 32$

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