HIGHER

8.9 Perpendicular Lines and the equation of a tangent: Test Pack

Total Marks

EDEXCEL 1MA1 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A circle has equation $x^2 + y^2 = 25$. Find the gradient of the tangent to the circle at the point (3, 4).

(Total for Question 1 is 1 mark)

2 For each gradient given below, write down the gradient of a line that is perpendicular to it.

(a) Gradient = 2 **(1)**

.....

(b) Gradient = -4 **(1)**

.....

(c) Gradient = 2/5 **(1)**

.....

(Total for Question 2 is 3 marks)

3 Which of these lines is perpendicular to y = -2x + 5?

- ☐ A) y = 2x - 3
- ☐ B) y = (1/2)x + 1
- ☐ C) y = -(1/2)x + 4
- ☐ D) y = -2x + 7

(Total for Question 3 is 1 mark)

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4 The circle $x^2 + y^2 = 25$ and the point P(4, 3) on it. The tangent at P meets the x-axis at A and the y-axis at B.

Work out the area of triangle OAB, where O is the origin.

(Total for Question 4 is 4 marks)

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5 A circle has centre O, the origin. For each point P given below, P lies on the circle, and OP is a radius.

Without finding the full equation of the tangent, state the gradient of the tangent to the circle at P.

(a) P = (4, 2) **(2)**

(b) P = (-6, 3) **(2)**

(c) P = (-5, -2) **(2)**

(Total for Question 5 is 6 marks)

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6 Show that for a circle $x^2 + y^2 = a^2$ the equation of the tangent at the point (x_0, y_0) on the circle is $x\,x_0 + y\,y_0 = a^2$.

(Total for Question 6 is 3 marks)

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7 A straight line L has equation $y = -x + 1$. Find an equation of the straight line that is perpendicular to L and passes through the point (-5, 4).

(Total for Question 7 is 3 marks)

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8 The point P(-7, 9) lies on the circle $x^2 + y^2 = 130$. Find an equation of the tangent to the circle at P.

(Total for Question 8 is 4 marks)

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9 A circle with centre O, the origin, has equation $x^2 + y^2 = 169$
The point (12, 5) lies on the circle.
Find the equation of the tangent to the circle at this point. Give your answer in the form $ax + by = c$, where a, b and c are integers.

(Total for Question 9 is 4 marks)

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10 A circle with centre O, the origin, has equation $x^2 + y^2 = 20$
P is the point (4, 2) and Q is the point (-4, -2). Both P and Q lie on the circle.
Prove that the tangent to the circle at P is parallel to the tangent to the circle at Q.

(Total for Question 10 is 4 marks)

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11 Priya is designing a circular pond in her garden. On a coordinate grid measured in metres, the pond has centre O, the origin, and its edge passes through the point P(9, 12). A straight path is to be built along the tangent to the pond's edge at P.

(a) Find the equation of the path. Give your answer in the form ax + by = c, where a, b and c are integers. **(4)**

(b) The main garden path runs along the x-axis. Find the coordinates of the point where Priya's new path crosses the main garden path. **(2)**

(Total for Question 11 is 6 marks)

12 A circle with centre O, the origin, has equation $x^2 + y^2 = 169$
The line with equation $5x + 12y = 169$ is a tangent to this circle.
Show that the point of contact of the tangent with the circle is (5, 12).

(Total for Question 12 is 4 marks)

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Mark scheme · 8.9 Perpendicular Lines and the equation of a tangent: Test Pack

Question 1

gradient = -3/4 cao **B1**

Answer: -3/4 (or -0.75)

Question 2

(a) -1/2 (oe -0.5) (cao) **B1**

(a) Answer: -1/2

(b) 1/4 (oe 0.25) (cao) **B1**

(b) Answer: 1/4

(c) -5/2 (oe -2.5) (cao) **B1**

(c) Answer: -5/2

Question 3

B selected (cao) **B1**

Answer: B (y = (1/2)x + 1)

Question 4

find tangent equation: slope -4/3 and intercept c with 3 = -4/3*4 + c so c = 25/3 oe **M1**

find x-intercept A: 0 = -4/3 x + 25/3 so x = 25/4 and y-intercept B: (0, 25/3) oe **M1**

area formula 1/2 * x_intercept * y_intercept oe **M1**

area = 1/2 * 25/4 * 25/3 = 625/24 cao **A1**

Answer: 625/24

Question 5

(a) gradient of OP = 2/4 = 1/2 (oe) **M1**

(a) -2 (cao) **A1**

(a) Answer: -2

(b) gradient of OP = 3/(-6) = -1/2 (oe) **M1**

(b) 2 (cao) **A1**

(b) Answer: 2

(c) gradient of OP = -2/-5 = 2/5 (oe) **M1**

(c) -5/2 (oe -2.5) (cao) **A1**

(c) Answer: -5/2

Question 6

use the radius vector (x0, y0) is perpendicular to tangent so (x0)(x - x0) + (y0)(y - y0) = 0 oe **dM1**

expand or rearrange to get x x0 + y y0 = $x0^2 + y0^2$ oe **M1**

substitute $x0^2 + y0^2 = a^2$ to give x x0 + y y0 = a^2 cso **A1**

Answer: $x\,x_0 + y\,y_0 = a^2$

Question 7

gradient of the perpendicular = -1/(-1) = 1 **M1**

substitutes (-5, 4) to find the intercept **M1**

y = x + 9 oe **A1**

Answer: y = x + 9

Question 8

gradient of OP = -9/7 **M1**

gradient of the tangent = -1/(-9/7) = 7/9 **M1**

substitutes (-7, 9) into y = mx + c **M1**

y = (7/9)x + 130/9 oe **A1**

Answer: y = (7/9)x + 130/9

Question 9

gradient of radius = 5/12 **M1**

tangent gradient = -12/5 (negative reciprocal of 5/12) **M1**

correct method using the point (12, 5), e.g. y - 5 = -(12/5)(x - 12) **M1**

12x + 5y = 169 (oe, integer coefficients) (cao) **A1**

Answer: 12x + 5y = 169

Question 10

gradient of OP = 2/4 = 1/2, so tangent gradient at P = -2 **M1**

gradient of OQ = (-2)/(-4) = 1/2, so tangent gradient at Q = -2 **M1**

both tangent gradients equal -2 **A1**

correct conclusion: since the tangents have equal gradients and touch the circle at different points, they are parallel (not the same line), as required (cso) **C1**

Answer: Proved: both tangents have gradient -2, so they are parallel

Question 11

(a) recognises OP = $\sqrt{9^2 + 12^2}$ = 15, so the equation of the pond's edge is $x^2 + y^2 = 225$ **M1**

(a) gradient of OP = 12/9 = 4/3, so tangent gradient = -3/4 **M1**

(a) correct method using point P, e.g. y - 12 = -(3/4)(x - 9) **M1**

(a) 3x + 4y = 75 (oe, integer coefficients) (cao) **A1**

(a) Answer: 3x + 4y = 75

(b) sets y = 0 in their equation from part (a), ft **M1**

(b) (25, 0) (cao, ft) **A1**

(b) Answer: (25, 0)

Question 12

checks $5^2 + 12^2 = 169$, so (5, 12) lies on the circle **B1**

checks 5(5) + 12(12) = 169, so (5, 12) lies on the tangent line **B1**

checks perpendicularity: gradient of the radius to (5, 12) is 12/5, and the gradient of the line 5x + 12y = 169 is -5/12, and (12/5) x (-5/12) = -1 **M1**

correct conclusion: (5, 12) lies on both the circle and the line, and the radius to this point is perpendicular to the line, so (5, 12) must be the point of contact (cso) **C1**

Answer: (5, 12) is the point of contact (shown)

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