HIGHER

8.9D Perpendicular Lines and the equation of a tangent: Fluency and Exam Drill

Total Marks

EDEXCEL Edexcel 1MA1 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A circle has equation $x^2 + y^2 = 25$. Find the gradient of the tangent to the circle at the point (3, 4).

(Total for Question 1 is 1 mark)

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2 For the circle $x^2 + y^2 = 25$, write down the equation of the tangent at the point (3, 4).

(Total for Question 2 is 2 marks)

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3 Find the equation of the tangent to the circle $x^2 + y^2 = 625$ at the point (7, 24).

(Total for Question 3 is 2 marks)

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4 The circle has equation $(x - 1)^2 + (y + 2)^2 = 25$ and P is the point (5, 1) on the circle.
Find the equation of the line through P that is perpendicular to the tangent at P.

(Total for Question 4 is 2 marks)

5 The circle $x^2 + y^2 = 50$ meets the vertical line $x = 5$ at two points. Take the point with positive y-coordinate.
Work out the equation of the tangent to the circle at this point.

(Total for Question 5 is 3 marks)

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6 Show that for a circle $x^2 + y^2 = a^2$ the equation of the tangent at the point (x_0, y_0) on the circle is $x\,x_0 + y\,y_0 = a^2$.

(Total for Question 6 is 3 marks)

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7 The circle $x^2 + y^2 = 100$ has a point A with x-coordinate 8 and y positive.
(a) Find the equation of the tangent at A.
(b) Find the coordinates of the point where this tangent meets the line $y = 2x - 5$.

(a) Find the equation of the tangent at A. **(2)**

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(b) Find the coordinates of the intersection point of the tangent and the line $y = 2x - 5$. **(2)**

.....

(Total for Question 7 is 4 marks)

8 Circle C has equation $(x - 1)^2 + (y - 2)^2 = 25$ and point P is (4, 6) on C.
(a) Find the equation of the tangent at P.
(b) Find the equation of the line through P that is perpendicular to this tangent.

(a) Find the equation of the tangent at P. **(2)**

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(b) Find the equation of the line through P perpendicular to the tangent. **(2)**

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(Total for Question 8 is 4 marks)

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9 For the circle $x^2 + y^2 = 25$ find the point or points where the tangent has gradient 2.

(Total for Question 9 is 3 marks)

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10 The circle $x^2 + y^2 = 25$ and the point P(4, 3) on it. The tangent at P meets the x-axis at A and the y-axis at B.
Work out the area of triangle OAB, where O is the origin.

(Total for Question 10 is 4 marks)

11 A circle has equation $x^2 + y^2 = 50$ and a point T lies on the circle. Show that the tangents from the external point (10, 0) touch the circle at points with x-coordinate 5, and find the coordinates of these points of contact.

(Total for Question 11 is 4 marks)

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12 The circle $x^2 + y^2 = 169$ is given and the line L has equation $3x - 4y + 10 = 0$.
(a) Find the points on the circle where the tangent is perpendicular to L.
(b) Find the equations of the tangents at those points.

(a) Find the points on the circle where the tangent is perpendicular to the line L. **(4)**

(b) Find the equations of the tangents at those points. **(4)**

(Total for Question 12 is 8 marks)

Mark scheme · 8.9D Perpendicular Lines and the equation of a tangent: Fluency and Exam Drill

Question 1

- **B1** gradient = -3/4 cao
- **Answer: -3/4 (or -0.75)**

Question 2

- **M1** use tangent gradient -3/4 and substitute point to find intercept oe
- **A1** y = -3/4 x + 25/4 cao
- **Answer: y = -3/4 x + 25/4**

Question 3

- **M1** use $xx0 + yy0 = r^2$ or find slope -7/24 then form equation oe
- **A1** 7x + 24y = 625 cao
- **Answer: 7x + 24y = 625**

Question 4

- **M1** find gradient of radius = (1 - (-2)) / (5 - 1) = 3/4 or identify perpendicular slope oe
- **A1** equation y - 1 = 3/4 (x - 5) or y = 3/4 x - 11/4 cao
- **Answer: y = 3/4 x - 11/4**

Question 5

- **M1** find y: $y^2 = 50 - 25$ so y = 5 (positive) oe
- **M1** find tangent gradient -x/y = -5/5 = -1 oe
- **A1** y = -x + 10 cao
- **Answer: y = -x + 10**

Question 6

- **dM1** use the radius vector (x0, y0) is perpendicular to tangent so (x0)(x - x0) + (y0)(y - y0) = 0 oe
- **M1** expand or rearrange to get $x\,x0 + y\,y0 = x0^2 + y0^2$ oe
- **A1** substitute $x0^2 + y0^2 = a^2$ to give $x\,x0 + y\,y0 = a^2$ cso
- **Answer: $x\,x_0 + y\,y_0 = a^2$**

Question 7

- (a) **M1** find y: $y^2 = 100 - 64$ so y = 6 and slope -x/y = -8/6 = -4/3 oe
- (a) **A1** y = -4/3 x + 50/3 cao
- **(a) Answer: y = -4/3 x + 50/3**
- (b) **M1** solve -4/3 x + 50/3 = 2x - 5 oe
- (b) **A1** intersection (13/2, 8) cao
- **(b) Answer: (13/2, 8)**

Question 8

- (a) **M1** radius slope = (6 - 2)/(4 - 1) = 4/3 so tangent slope = -3/4 oe
- (a) **A1** y = -3/4 x + 9 cao
- **(a) Answer: y = -3/4 x + 9**
- (b) **M1** perpendicular slope = 4/3 and form y - 6 = 4/3 (x - 4) oe
- (b) **A1** y = 4/3 x + 2/3 cao
- **(b) Answer: y = 4/3 x + 2/3**

Question 9

- **M1** set tangent gradient -x/y = 2 so x = -2y oe
- **M1** substitute: $(-2y)^2 + y^2 = 25$ so $5y^2 = 25$ so $y^2 = 5$ oe
- **A1** points $(-2\sqrt{5}, \sqrt{5})$ and $(2\sqrt{5}, -\sqrt{5})$ cao
- **Answer: $(-2\sqrt{5}, \sqrt{5})$ and $(2\sqrt{5}, -\sqrt{5})$**

Question 10

- **M1** find tangent equation: slope -4/3 and intercept c with 3 = -4/3*4 + c so c = 25/3 oe
- **M1** find x-intercept A: 0 = -4/3 x + 25/3 so x = 25/4 and y-intercept B: (0, 25/3) oe
- **M1** area formula 1/2 * x_intercept * y_intercept oe
- **A1** area = 1/2 * 25/4 * 25/3 = 625/24 cao
- **Answer: 625/24**

Question 11

- **M1** write tangent at (x0,y0) as x x0 + y y0 = 50 and substitute (10,0): 10 x0 = 50 oe
- **M1** so x0 = 5 and use circle to find $y0^2 = 50 - 25 = 25$ oe
- **M1** find y0 = ±5 oe
- **A1** points (5,5) and (5,-5) cao
- **Answer: (5, 5) and (5, -5)**

Question 12

- (a) **M1** find gradient of L: 3x - 4y + 10 = 0 gives y = 3/4 x + 5/2 so gradient 3/4; perpendicular gradient -4/3 oe
- (a) **M1** set tangent gradient -x0/y0 = -4/3 so x0/y0 = 4/3 so x0 = 4k, y0 = 3k oe
- (a) **M1** substitute into circle: $(4k)^2 + (3k)^2 = 169$ so $25k^2 = 169$ so k = ±13/5 oe
- (a) **A1** points (52/5, 39/5) and (-52/5, -39/5) cao
- **(a) Answer: (52/5, 39/5) and (-52/5, -39/5)**
- (b) **M1** tangent gradient is -4/3 so form y = -4/3 x + c and substitute (52/5,39/5) to find c: 39/5 = -4/3 * 52/5 + c oe
- (b) **M1** compute c: multiply by 15 => 117 = -208 + 15c so 15c = 325 so c = 65/3 oe
- (b) **M1** other tangent through (-52/5,-39/5) gives c = -65/3 by symmetry oe
- (b) **A1** equations y = -4/3 x + 65/3 and y = -4/3 x - 65/3 cao
- **(b) Answer: y = -4/3 x + 65/3 and y = -4/3 x - 65/3**