HIGHER

H06-30min The Normal Distribution: 30-minute Pack

Total Marks

EDEXCEL 1ST0 · Calculator allowed · about 30 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The lengths of steel rods produced by a machine are normally distributed with a mean of 250 cm and a standard deviation of 3 cm.

(a) Work out the interval within which approximately 68% of the rods' lengths lie. **(2)**

.....

(b) Work out the interval within which approximately 95% of the rods' lengths lie. **(2)**

.....

(Total for Question 1 is 4 marks)

2 The resting heart rates of adults in a large population are normally distributed with a mean of 72 beats per minute (bpm) and a standard deviation of 6 bpm.

(a) Write down the percentage of adults with a resting heart rate between 60 bpm and 84 bpm. **(2)**

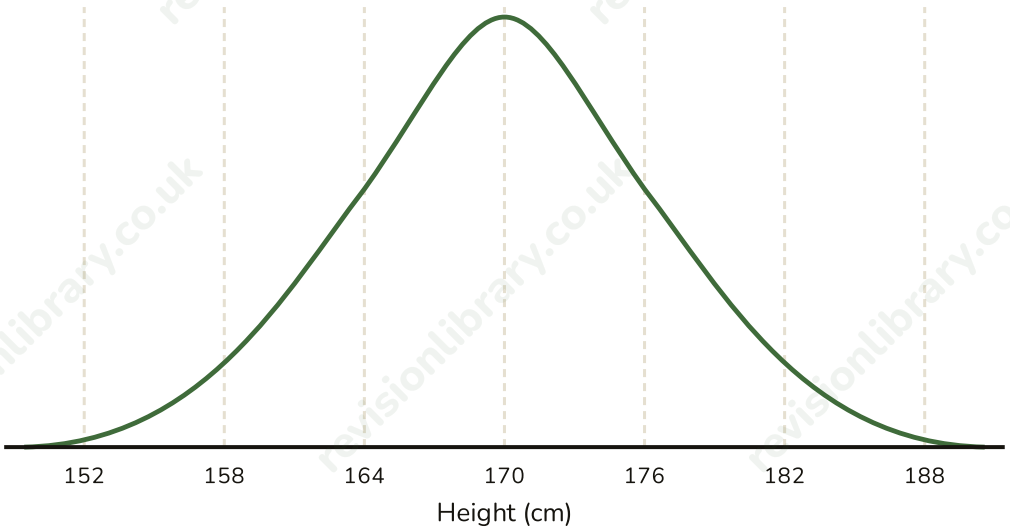
.....

(b) Work out the percentage of adults with a resting heart rate above 90 bpm. **(3)**

.....

(Total for Question 2 is 5 marks)

3 The heights of a large group of adult cyclists are normally distributed with a mean of 170 cm and a standard deviation of 6 cm. The diagram shows the distribution, with several height values marked on the horizontal axis.



(a) State how many standard deviations the height 158 cm is from the mean, and in which direction. **(1)**

(b) State how many standard deviations the height 176 cm is from the mean, and in which direction. **(1)**

(c) State how many standard deviations the height 188 cm is from the mean, and in which direction. **(1)**

(d) Using the empirical rule, write down the percentage of cyclists with a height between 164 cm and 188 cm. **(1)**

.....

(Total for Question 3 is 4 marks)

4 A factory produces bags of flour with a labelled mass of 1 kg (1000 g). Before setting tolerance limits, a quality inspector wants to check whether the masses of the bags are approximately normally distributed. This investigation follows the statistical enquiry cycle: plan, collect, process and interpret.

(a) PLAN: Describe a suitable method the inspector could use to select a sample of 50 bags from a day's production for weighing, and explain how this method helps to avoid bias. **(2)**

(b) COLLECT/PROCESS: The inspector weighs a small sample of 8 bags. Their masses, in grams, are shown in the table. **(2)**

Bag	1	2	3	4	5	6	7	8
Mass (g)	994	998	1001	1002	999	1005	997	1000

Work out the mean mass of this sample.

.....

(c) PROCESS: Work out the median mass of the sample of 8 bags in part (b). **(1)**

.....

(d) INTERPRET: A much larger sample confirms that the masses of all bags produced are normally distributed with a mean of 1000 g and a standard deviation of 8 g. The factory sets tolerance limits of 984 g to 1016 g, and rejects any bag outside these limits. Use the empirical rule to estimate the percentage of bags that would be rejected, and comment on whether the tolerance limits seem reasonable. **(3)**

(Total for Question 4 is 8 marks)

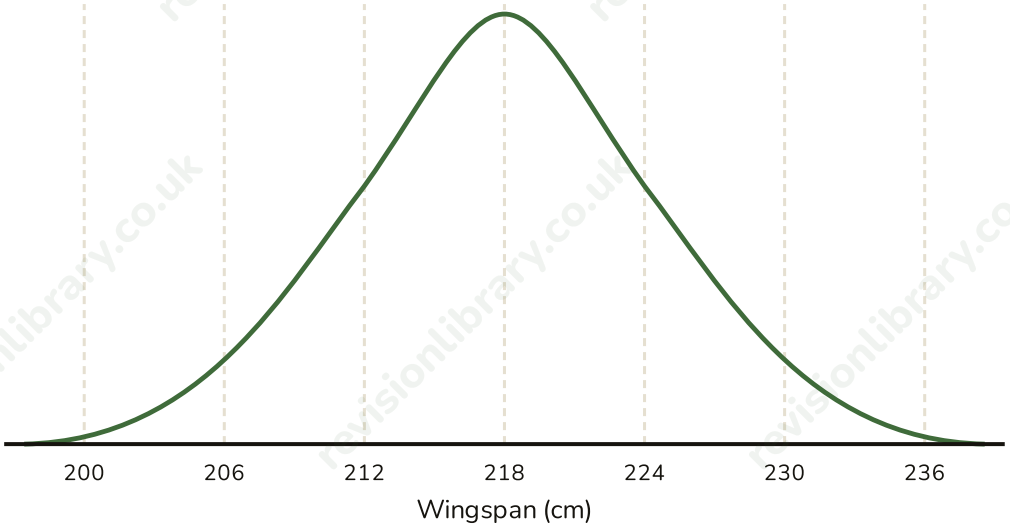
5 The times taken by runners to complete a marathon are normally distributed with a mean of 240 minutes and a standard deviation of 20 minutes.

(a) 210 minutes is 1.5 standard deviations below the mean, and 270 minutes is 1.5 standard deviations above the mean. Since 1.5 standard deviations is halfway between 1 and 2 standard deviations, use linear interpolation between the known percentages for 1 standard deviation (68%) and 2 standard deviations (95%) to estimate the percentage of runners with a time between 210 and 270 minutes. **(3)**

(b) Out of 3000 entrants, estimate the number of runners whose time is outside the interval 210 to 270 minutes. **(2)**

(Total for Question 5 is 5 marks)

6 The wingspans of a large population of adult swans are normally distributed with a mean of 218 cm and a standard deviation of 6 cm. The diagram shows the distribution, with several wingspan values marked on the horizontal axis.



(a) Using the diagram, write down the interval of wingspans within which approximately 95% of the swans lie. **(2)**

.....

(b) A swan has a wingspan of 227 cm. This corresponds to a standardised score of $z = 1.5$. Show that this value of z is correct. Then, using linear interpolation between the known percentages for 1 standard deviation (68%) and 2 standard deviations (95%), estimate the percentage of swans with a wingspan greater than 227 cm. **(3)**

(c) Give one reason why the normal distribution may not be a perfectly accurate model for the wingspans of swans. **(1)**

(Total for Question 6 is 6 marks)

Mark scheme · H06-30min The Normal Distribution: 30-minute Pack

Question 1

- (a) **M1** 250-3 and 250+3, oe
- (a) **A1** 247 cm to 253 cm cao
- **(a) Answer: 247 cm to 253 cm**
- (b) **M1** 250-(2x3) and 250+(2x3), oe
- (b) **A1** 244 cm to 256 cm cao
- **(b) Answer: 244 cm to 256 cm**

Question 2

- (a) **M1** identifies 60 and 84 as 2 standard deviations from the mean, oe
- (a) **A1** 95% cao
- **(a) Answer: 95%**
- (b) **M1** identifies 90 as 3 standard deviations above the mean, oe
- (b) **M1** (100-99.7)/2, oe
- (b) **A1** 0.15% cao
- **(b) Answer: 0.15%**

Question 3

- (a) **B1** 2 standard deviations below the mean, cao
- **(a) Answer: 2 standard deviations below the mean**
- (b) **B1** 1 standard deviation above the mean, cao
- **(b) Answer: 1 standard deviation above the mean**
- (c) **B1** 3 standard deviations above the mean, cao
- **(c) Answer: 3 standard deviations above the mean**
- (d) **B1** 83.85% cao
- **(d) Answer: 83.85%**

Question 4

- (a) **B1** suitable sampling method described, e.g. a systematic sample, selecting every nth bag as they come off the production line
- (a) **B1** valid reason linked to avoiding bias, e.g. this spreads the sample across the whole day's production, rather than only bags made at one time, so it better represents the day's output
- **(a) Answer: e.g. Take a systematic sample, selecting every nth bag from the production line throughout the day; this avoids bias because bags from all points in the day's production are included, not just bags made at one time.**
- (b) **M1** 994+998+1001+1002+999+1005+997+1000 (=7996), oe
- (b) **A1** 999.5 g cao
- **(b) Answer: 999.5 g**
- (c) **B1** 999.5 g cao
- **(c) Answer: 999.5 g**
- (d) **M1** identifies 984 and 1016 as 2 standard deviations from the mean, oe
- (d) **A1** 5% cao (100-95)
- (d) **B1** valid comment, e.g. the limits seem reasonable since only a small proportion (about 5%) of bags would fall outside them
- **(d) Answer: About 5% of bags would be rejected; the tolerance limits seem reasonable since only a small proportion of bags fall outside them.**

Question 5

- (a) **M1** recognises the required percentage is halfway between 68% and 95%, oe
- (a) **M1** 68+(95-68)/2, oe
- (a) **A1** 81.5% cao
- **(a) Answer: 81.5%**
- (b) **M1** 100-81.5 (=18.5%), oe
- (b) **A1** 555 cao
- **(b) Answer: 555**

Question 6

- (a) **M1** 218-2(6) and 218+2(6), oe
- (a) **A1** 206 cm to 230 cm cao
- **(a) Answer: 206 cm to 230 cm**
- (b) **M1** (227-218)/6=1.5, shown
- (b) **M1** 68+(95-68)/2 (=81.5) used as the percentage within 1.5 sd
- (b) **A1** 9.25% cao (=(100-81.5)/2)
- **(b) Answer: 9.25%**
- (c) **B1** valid reason, e.g. wingspan cannot be negative, but the normal distribution extends infinitely in both directions, so the model cannot be perfectly accurate at extreme (low) values
- **(c) Answer: e.g. A swan's wingspan cannot be negative, but the normal distribution curve theoretically extends infinitely in both directions, so it cannot be a perfectly accurate model at the extremes.**