



H07-30min The Binomial Distribution: 30-minute Pack

EDEXCEL 1ST0 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / ____

Answer ALL questions. Show all your working.

- 1** A biased coin is designed so that $P(\text{Heads}) = 0.3$ on each throw. The coin is thrown 6 times. Let X be the number of heads obtained.

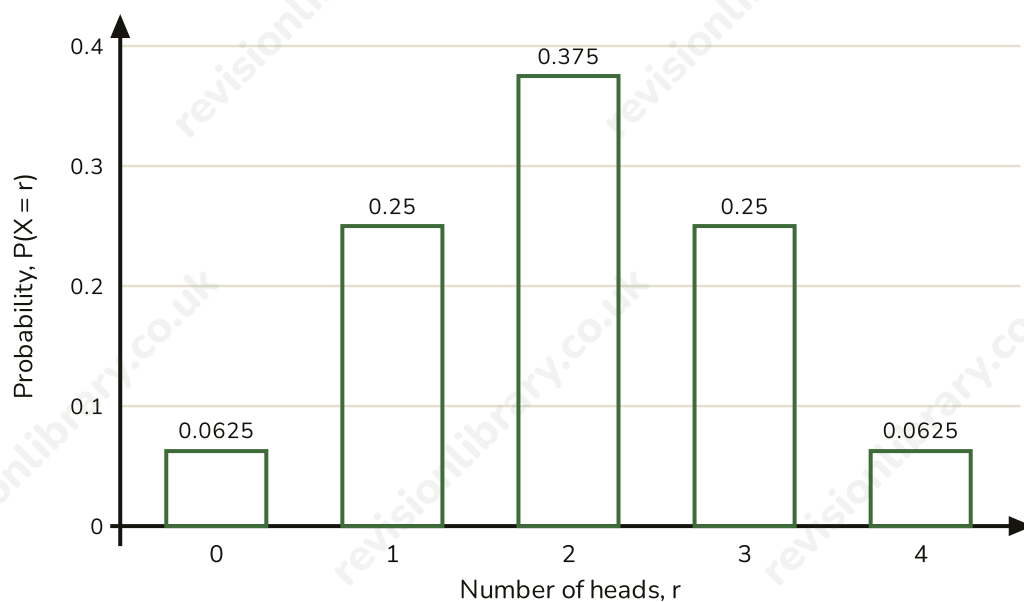
(a) Write down the value of n and the value of p . (2)

(b) Use the formula $P(X = r) = {}^n C_r p^r (1 - p)^{n-r}$ to find $P(X = 2)$. (3)

(c) Find $P(X = 0)$. (2)

(Total for Question 1 is 7 marks)

- 2 A fair coin is tossed 4 times. Let X be the number of heads obtained. The bar chart shows the probability distribution of X .



- (a) Write down $P(X = 3)$. (1)

- (b) Find $P(X \leq 1)$, the probability of at most one head. (2)

- (c) Show, using the formula $P(X = r) = nCr p^r (1 - p)^{n-r}$, that $P(X = 2) = 0.375$. (2)

(Total for Question 2 is 5 marks)

3 A factory finds that, on average, 5% of the light bulbs it produces are faulty. A random sample of 10 bulbs is selected for testing. Let X be the number of faulty bulbs in the sample.

(a) Write down the value of n and the value of p . (2)

(b) Find $P(X = 2)$, the probability that exactly 2 bulbs in the sample are faulty. (3)

(c) Find the probability that at least one bulb in the sample is faulty. (3)

(Total for Question 3 is 8 marks)

4 A quality-control officer at a factory is investigating whether faulty coat hangers are still being produced at the same rate as before a machine setting was changed. Historically, 8% of coat hangers produced were faulty. She plans to take a random sample of 12 coat hangers from today's production and record X , the number of faulty hangers in the sample.

(a) State two conditions that must hold in this context for X to be modelled by the distribution $B(12, 0.08)$. (2)

(b) Assuming $X \sim B(12, 0.08)$, find $P(X = 0)$, the probability that the sample contains no faulty hangers. (2)

(c) The officer collects her sample and finds that 3 of the 12 coat hangers are faulty. Using $B(12, 0.08)$, find $P(X = 3)$. (2)

(d) Using your answer to part (c), comment on whether the sample provides evidence that the proportion of faulty hangers has increased above 8%. (1)

(Total for Question 4 is 7 marks)

5 A machine fills bottles of sauce. Let p be the probability that a randomly chosen bottle is underfilled. In a batch of 40 bottles, the mean number of underfilled bottles is 3.

(a) Show that $p = 0.075$. (2)

(b) Using $p = 0.075$, find the probability that a batch of 40 bottles contains exactly 2 underfilled bottles. (3)

(Total for Question 5 is 5 marks)

Mark scheme · H07-30min The Binomial Distribution: 30-minute Pack

Question 1

- (a) **B1** $n = 6$ cao
- (a) **B1** $p = 0.3$ cao
- (a) **Answer: $n = 6, p = 0.3$**
- (b) **M1** $6C2 = 15$ identified
- (b) **M1** $15 \times 0.3^2 \times 0.7^4$, oe substitution
- (b) **A1** awrt 0.324
- (b) **Answer: 0.324 (awrt)**
- (c) **M1** 0.7^6 , oe (6C0 = 1 identified)
- (c) **A1** awrt 0.118
- (c) **Answer: 0.118 (awrt)**

Question 2

- (a) **B1** 0.25 cao
- (a) **Answer: 0.25**
- (b) **M1** $0.0625 + 0.25$, oe (adds the bars for $r = 0$ and $r = 1$)
- (b) **A1** 0.3125 oe cao
- (b) **Answer: 0.3125**
- (c) **M1** $4C2 = 6$ identified and correct substitution $6 \times 0.5^2 \times 0.5^2$
- (c) **A1** shown to equal 0.375, matching the chart
- (c) **Answer: 0.375**

Question 3

- (a) **B1** $n = 10$ cao
- (a) **B1** $p = 0.05$ cao
- (a) **Answer: $n = 10, p = 0.05$**
- (b) **M1** $10C2 = 45$ identified
- (b) **M1** $45 \times 0.05^2 \times 0.95^8$, oe substitution
- (b) **A1** awrt 0.0746
- (b) **Answer: 0.0746 (awrt)**
- (c) **M1** recognises $P(\text{at least one}) = 1 - P(X = 0)$
- (c) **M1** $P(X = 0) = 0.95^{10}$, awrt 0.599
- (c) **A1** awrt 0.401
- (c) **Answer: 0.401 (awrt)**

Question 4

- (a) **B1** each hanger's fault status is independent of the others
- (a) **B1** the probability that a hanger is faulty is the same (8%) for every hanger in the sample
- (a) **Answer: Each hanger's outcome is independent of the others; the probability of being faulty stays at 8% for every hanger.**
- (b) **M1** 0.92^{12} , oe $(12C0 = 1 \text{ identified})$
- (b) **A1** awrt 0.368
- (b) **Answer: 0.368 (awrt)**
- (c) **M1** $12C3 = 220$ and correct substitution $220 \times 0.08^3 \times 0.92^9$
- (c) **A1** awrt 0.0532
- (c) **Answer: 0.0532 (awrt)**
- (d) **B1** valid interpretation, e.g. since $P(X = 3)$ is small (about 5%), obtaining 3 faulty hangers would be unusual if p were still 0.08, so this provides some evidence that the true proportion has increased
- (d) **Answer: $P(X = 3)$ is only about 0.053 (roughly 5%), so getting 3 faulty hangers would be unlikely if p were still 0.08. This gives some evidence that the proportion of faulty hangers may have increased.**

Question 5

- (a) **M1** sets up $E(X) = np$, i.e. $40p = 3$
- (a) **A1** $p = 3/40 = 0.075$ shown
- (a) **Answer: 0.075**
- (b) **M1** $40C2 = 780$ identified
- (b) **M1** $780 \times 0.075^2 \times 0.925^{38}$, oe substitution
- (b) **A1** awrt 0.227
- (b) **Answer: 0.227 (awrt)**