Further Pure Maths: Radians and Trigonometry

EDEXCEL 4PM1 · Calculator allowed · about 65 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Show your working. Use radians or degrees as stated in each question. Give exact answers unless a degree of accuracy is specified. Questions marked non-calculator should be attempted without a calculator. Accept equivalent exact forms and valid alternative methods for the corresponding marks.

1 Solve $2*\sin(x)^2 - 3*\sin(x) + 1 = 0$ for $0 \le x < 2*\pi$. Give every solution exactly in radians.

(Total for Question 1 is 4 marks)

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2 Use a compound-angle formula to find the exact value of cos(75 degrees). Show your working.

(Total for Question 2 is 3 marks)

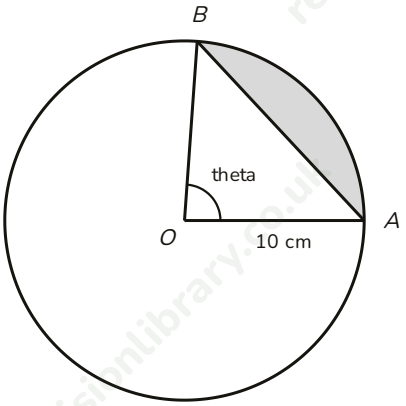
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3 Angles a and b are acute. Given sin(a) = 5/13 and cos(b) = 8/17, find the exact values of sin(a + b) and cos(a - b). Show your working.

(Total for Question 3 is 5 marks)

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4 A minor sector OAB has radius 10 cm and perimeter 35 cm, including OA, OB and the minor arc AB. A straight chord joins A to B. Find the angle AOB in radians, the area of the minor segment between the chord and the arc to 3 significant figures, and the length of chord AB to 3 significant figures.



(Total for Question 4 is 6 marks)

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5 Starting with the compound-angle formula for sin(x + x), solve sin(2*x) = cos(x) for $0 \leq x < 2*\pi$. Give all solutions exactly in radians and show how you avoid losing any solutions.

(Total for Question 5 is 5 marks)

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6 An arc of a circle of radius 9 cm subtends an angle of 1.4 radians at the centre. Calculate the length of the arc.

(Total for Question 6 is 2 marks)

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7 The perimeter of a sector is 28 cm. Its radius is 7 cm. Find its angle at the centre in radians and its area.

(Total for Question 7 is 3 marks)

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8 In triangle ABC, AB = 7 cm, AC = 10 cm and angle BAC = 1.2 radians. Calculate BC, giving your answer to 3 significant figures.

(Total for Question 8 is 3 marks)

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9 In triangle PQR, angle PQR = 42 degrees, angle PRQ = 73 degrees and PR = 9 cm. Calculate PQ to 3 significant figures.

(Total for Question 9 is 3 marks)

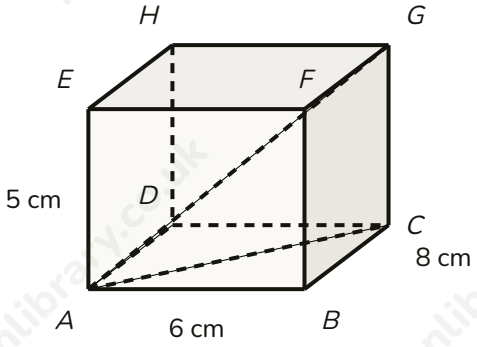
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10 Triangle DEF has DE = 8 cm, EF = 11 cm and DF = 14 cm. Calculate its largest angle to 1 decimal place, then its area to 3 significant figures.

(Total for Question 10 is 5 marks)

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11 A cuboid has rectangular base ABCD with AB = 6 cm and BC = 8 cm. E, F, G and H are vertically above A, B, C and D respectively, and CG = 5 cm. Find the exact length of AG and the angle AG makes with the plane ABCD, to 1 decimal place.



(Total for Question 11 is 4 marks)

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Mark scheme · Further Pure Maths: Radians and Trigonometry

Question 1

Factor to (2*sin(x) - 1)*(sin(x) - 1) = 0, or solve the quadratic in sin(x).	**M1**
Obtain sin(x) = 1/2 and sin(x) = 1.	**A1**
π/6 and 5*π/6 from sin(x) = 1/2.	**A1**
π/2 from sin(x) = 1, with no other solutions in the interval.	**A1**

Answer: x = π/6, π/2, 5*π/6

Question 2

Use cos(45 + 30) = cos(45)*cos(30) - sin(45)*sin(30), or another valid compound-angle decomposition.	**M1**
Substitute the appropriate exact trigonometric values.	**M1**
$(\sqrt{6} - \sqrt{2})/4$; accept equivalent exact forms.	**A1**

Answer: $(\sqrt{6} - \sqrt{2})/4$

Question 3

Obtain cos(a) = 12/13 and sin(b) = 15/17, using positivity for acute angles.	**B1**
Use sin(a+b) = sin(a)*cos(b) + cos(a)*sin(b).	**M1**
sin(a+b) = 220/221; accept equivalent exact forms.	**A1**
Use cos(a-b) = cos(a)*cos(b) + sin(a)*sin(b).	**M1**
cos(a-b) = 171/221; accept equivalent exact forms.	**A1**

Answer: sin(a + b) = 220/221; cos(a - b) = 171/221

Question 4

Subtract the two radii to obtain arc length 15 cm.	**M1**
Angle AOB = 15/10 = 1.5 radians.	**A1**
Calculate sector area (1/2)*100*1.5 and triangle area (1/2)*100*sin(1.5), using radians.	**M1**
Subtract triangle from sector: segment area = 25.1 cm^2 to 3 significant figures.	**A1**
Use chord = 2*10*sin(1.5/2), or cosine rule in triangle OAB.	**M1**
Chord AB = 13.6 cm to 3 significant figures.	**A1**

Answer: 1.5 radians; 25.1 cm^2; 13.6 cm

Question 5

Derive sin(2*x) = 2*sin(x)*cos(x) from the compound-angle formula.	**M1**
Rearrange and factor to cos(x)*(2*sin(x) - 1) = 0; alternatively separately consider cos(x)=0 before dividing.	**M1**
π/2 and 3*π/2 from cos(x) = 0.	**A1**
π/6 and 5*π/6 from sin(x) = 1/2.	**A1**
Give precisely the four solutions and explain that the cos(x)=0 cases must be kept when dividing by cos(x), or are retained by the zero-product method.	**A1**

Answer: x = π/6, π/2, 5*π/6, 3*π/2

Question 6

Use s = r*θ with θ in radians.	**M1**
12.6 cm; accept equivalent forms.	**A1**

Answer: 12.6 cm

Question 7

Subtract both radii to obtain arc length 14 cm.	**M1**
Angle = 2 radians.	**A1**
Area = 49 cm^2; accept equivalent forms.	**A1**

Answer: 2 radians; 49 cm^2

Question 8

Use $BC^2 = 7^2 + 10^2 - 2*7*10*\cos(1.2)$, in radian mode.	**M1**
Take the positive square root of their cosine-rule expression.	**M1**
9.91 cm to 3 significant figures; accept valid alternative methods.	**A1**

Answer: 9.91 cm

Question 9

Match PQ to 73 degrees and PR to 42 degrees in the sine rule.	**M1**
Rearrange to PQ = 9*sin(73 degrees)/sin(42 degrees).	**M1**
12.9 cm to 3 significant figures; accept valid alternative methods.	**A1**

Answer: 12.9 cm

Question 10

Identify angle DEF as opposite the longest side, DF.	**B1**
Use $\cos(DEF) = (8^2 + 11^2 - 14^2)/(2*8*11)$.	**M1**
93.6 degrees to 1 decimal place.	**A1**
Use area = (1/2)*8*11*sin(DEF), retaining an unrounded angle; accept Heron's formula or another valid method.	**M1**
43.9 cm^2 to 3 significant figures.	**A1**

Answer: 93.6 degrees; 43.9 cm^2

Question 11

Find the base diagonal AC = $\sqrt{6^2 + 8^2}$ = 10 cm.	**M1**
AG = $\sqrt{125}$ = $5*\sqrt{5}$ cm; accept equivalent exact forms.	**A1**
Identify the angle as GAC and use tan(GAC) = 5/10, or equivalent sine/cosine relation in triangle ACG.	**M1**
26.6 degrees to 1 decimal place.	**A1**

Answer: $5*\sqrt{5}$ cm; 26.6 degrees

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