Further Pure Maths: Integration and Definite Integrals: 30-minute Pack

EDEXCEL 4PM1 · Calculator allowed · about 30 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Give exact answers and show your working. In an indefinite integral, C is an arbitrary constant. In a definite integral, evaluate F(b) - F(a); the constant cancels, so the answer has no arbitrary constant.

1 A curve has $dy/dx = 6*x^2 - 4*x + 1$. It passes through the point (2, 7).

(a) Find the equation of the curve in the form y = f(x). **(3)**

(b) Using your equation from part (a), evaluate the definite integral, from x = 0 to x = 1, of y dx. **(3)**

(Total for Question 1 is 6 marks)

© Revision Library | revisionlibrary.co.uk

2 A machine deposits material at a rate of $3*t^2 - 2*t + 4$ grams per minute for $t \geq 0$. Let A(t) be the total mass deposited by time t minutes, including material already present. Initially $A(0) = 5/2$ grams. The increase in A over an interval is the integral of the rate over that interval.

(a) Find A(t), including the initial mass. **(3)**

(b) Find the mass added between $t = 1/2$ and $t = 5/2$. Evaluate the definite integral, from $t = 1/2$ to $t = 5/2$, of $3*t^2 - 2*t + 4$ dt. **(3)**

(Total for Question 2 is 6 marks)

3 Find the indefinite integral of $2*x^3 + x^2 - 6$ with respect to x.

(Total for Question 3 is 2 marks)

© Revision Library | revisionlibrary.co.uk

4 A curve has $dy/dx = 4*x^3 - 2*x + 3$. It passes through the point (1, 6). Find its equation in the form $y = f(x)$.

(Total for Question 4 is 3 marks)

---

5 Evaluate the definite integral, from $x = -1$ to $x = 2$, of $2*x^2 - 3*x + 4$ dx. Show the values at both limits.

(Total for Question 5 is 4 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · Further Pure Maths: Integration and Definite Integrals: 30-minute Pack

Question 1

(a) Integrate to $y = 2*x^3 - 2*x^2 + x + C$, allowing a coefficient error. **M1**

(a) Use (2, 7): $7 = 16 - 8 + 2 + C$, giving $C = -3$. Award for substituting the point into their integrated equation and solving for its constant. **M1**

(a) $y = 2*x^3 - 2*x^2 + x - 3$; accept equivalent forms. **A1**

(a) Answer: $y = 2*x^3 - 2*x^2 + x - 3$

(b) Integrate y term by term. An antiderivative is $F(x) = (1/2)*x^4 - (2/3)*x^3 + (1/2)*x^2 - 3*x$. **M1**

(b) Use $F(1) - F(0) = (1/2 - 2/3 + 1/2 - 3) - 0$. **M1**

(b) -8/3; accept equivalent exact forms. This is a signed integral. **A1**

(b) Answer: -8/3

Question 2

(a) Integrate the rate to $t^3 - t^2 + 4*t + C$, allowing a coefficient error. **M1**

(a) Use $A(0) = 5/2$ to obtain $C = 5/2$. Award for using the initial mass to determine the constant in their integrated expression. **M1**

(a) $A(t) = t^3 - t^2 + 4*t + 5/2$ grams; accept equivalent forms. **A1**

(a) Answer: $A(t) = t^3 - t^2 + 4*t + 5/2$ grams

(b) Use $[t^3 - t^2 + 4*t]$ from 1/2 to 5/2, or subtract $A(1/2)$ from $A(5/2)$. **M1**

(b) For the primitive without a constant, endpoint values are 155/8 and 15/8; alternatively $A(5/2) = 175/8$ and $A(1/2) = 35/8$. **A1**

(b) 35/2 grams added; accept equivalent exact forms. **A1**

(b) Answer: 35/2 grams

Question 3

Integrate term by term, including the constant term -6. **M1**

$(1/2)*x^4 + (1/3)*x^3 - 6*x + C$; accept equivalent forms. **A1**

Answer: $(1/2)*x^4 + (1/3)*x^3 - 6*x + C$

Question 4

Integrate to $y = x^4 - x^2 + 3*x + C$, allowing a coefficient error. **M1**

Use (1, 6): $6 = 1 - 1 + 3 + C$, giving $C = 3$. Award for substituting the point into their integrated equation and solving for its constant. **M1**

$y = x^4 - x^2 + 3*x + 3$; accept equivalent forms. **A1**

Answer: $y = x^4 - x^2 + 3*x + 3$

Question 5

Integrate term by term. An antiderivative is $F(x) = (2/3)*x^3 - (3/2)*x^2 + 4*x$. **M1**

$F(2) = 22/3$ and $F(-1) = -37/6$, using the displayed F. Accept consistent endpoint values from another antiderivative differing by a constant. **A1**

Subtract the lower-limit value from the upper-limit value: $22/3 - (-37/6)$. **M1**

27/2; accept equivalent exact forms. No arbitrary constant remains. **A1**

Answer: 27/2

© Revision Library | revisionlibrary.co.uk