Further Pure Maths: Integration and Definite Integrals

EDEXCEL 4PM1 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Give exact answers and show your working. In an indefinite integral, C is an arbitrary constant. In a definite integral, evaluate F(b) - F(a); the constant cancels, so the answer has no arbitrary constant.

1 Find the indefinite integral of $7*x^2$ with respect to x.

(Total for Question 1 is 1 mark)

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2 Water enters a container at a rate of $2*t^2 + 3$ litres per minute, where t is time in minutes. The volume added between t = 1 and t = 3 is the definite integral, from t = 1 to t = 3, of $2*t^2 + 3$ dt. Find this volume, showing both endpoint values.

(Total for Question 2 is 4 marks)

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3 A curve has $dy/dx = 6*x^2 - 4*x + 1$. It passes through the point (2, 7).

(a) Find the equation of the curve in the form y = f(x). **(3)**

(b) Using your equation from part (a), evaluate the definite integral, from x = 0 to x = 1, of y dx. **(3)**

(Total for Question 3 is 6 marks)

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4 A machine deposits material at a rate of $3*t^2 - 2*t + 4$ grams per minute for $t \geq 0$. Let A(t) be the total mass deposited by time t minutes, including material already present. Initially $A(0) = 5/2$ grams. The increase in A over an interval is the integral of the rate over that interval.

(a) Find A(t), including the initial mass. **(3)**

(b) Find the mass added between $t = 1/2$ and $t = 5/2$. Evaluate the definite integral, from $t = 1/2$ to $t = 5/2$, of $3*t^2 - 2*t + 4$ dt. **(3)**

(Total for Question 4 is 6 marks)

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5 Find the indefinite integral of 9 with respect to x.

(Total for Question 5 is 1 mark)

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6 Find the indefinite integral of $5*x^4 - 3*x$ with respect to x.

(Total for Question 6 is 2 marks)

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7 Find the indefinite integral of $2*x^3 + x^2 - 6$ with respect to x.

(Total for Question 7 is 2 marks)

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8 A curve has $dy/dx = 4*x^3 - 2*x + 3$. It passes through the point (1, 6). Find its equation in the form $y = f(x)$.

(Total for Question 8 is 3 marks)

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9 A curve has $dy/dx = 3*x^2 + 4*x - 5$. It passes through the point (-1, 2). Find its equation in the form $y = f(x)$.

(Total for Question 9 is 3 marks)

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10 Evaluate the definite integral, from $x = -1$ to $x = 2$, of $2*x^2 - 3*x + 4$ dx. Show the values at both limits.

(Total for Question 10 is 4 marks)

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11 Evaluate the definite integral, from x = 1/2 to x = 3/2, of $6*x^2 + 2*x - 1$ dx. Show the values at both limits.

(Total for Question 11 is 4 marks)

12 Let $f(x) = 3*x^2 - 2*x + 4$. Use C for the arbitrary constant in an indefinite integral; in a definite integral the constants cancel.

(a) Find the indefinite integral of f(x) with respect to x. **(2)**

(b) Evaluate the definite integral, from x = 0 to x = 2, of $3*x^2 - 2*x + 4$ dx. **(2)**

(Total for Question 12 is 4 marks)

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Mark scheme · Further Pure Maths: Integration and Definite Integrals

Question 1

$(7/3)*x^3 + C$, where C is an arbitrary constant; accept equivalent forms. **B1**

Answer: $(7/3)*x^3 + C$

Question 2

Integrate the inflow rate. An antiderivative is $F(t) = (2/3)*t^3 + 3*t$. **M1**

$F(3) = 27$ and $F(1) = 11/3$, using the displayed F. Accept consistent endpoint values from another antiderivative differing by a constant. **A1**

Calculate $F(3) - F(1) = 27 - 11/3$. **M1**

70/3 litres; accept equivalent exact forms. **A1**

Answer: 70/3 litres

Question 3

(a) Integrate to $y = 2*x^3 - 2*x^2 + x + C$, allowing a coefficient error. **M1**

(a) Use (2, 7): $7 = 16 - 8 + 2 + C$, giving $C = -3$. Award for substituting the point into their integrated equation and solving for its constant. **M1**

(a) $y = 2*x^3 - 2*x^2 + x - 3$; accept equivalent forms. **A1**

(a) Answer: $y = 2*x^3 - 2*x^2 + x - 3$

(b) Integrate y term by term. An antiderivative is $F(x) = (1/2)*x^4 - (2/3)*x^3 + (1/2)*x^2 - 3*x$. **M1**

(b) Use $F(1) - F(0) = (1/2 - 2/3 + 1/2 - 3) - 0$. **M1**

(b) -8/3; accept equivalent exact forms. This is a signed integral. **A1**

(b) Answer: -8/3

Question 4

(a) Integrate the rate to $t^3 - t^2 + 4*t + C$, allowing a coefficient error. **M1**

(a) Use $A(0) = 5/2$ to obtain $C = 5/2$. Award for using the initial mass to determine the constant in their integrated expression. **M1**

(a) $A(t) = t^3 - t^2 + 4*t + 5/2$ grams; accept equivalent forms. **A1**

(a) Answer: $A(t) = t^3 - t^2 + 4*t + 5/2$ grams

(b) Use $[t^3 - t^2 + 4*t]$ from 1/2 to 5/2, or subtract $A(1/2)$ from $A(5/2)$. **M1**

(b) For the primitive without a constant, endpoint values are 155/8 and 15/8; alternatively $A(5/2) = 175/8$ and $A(1/2) = 35/8$. **A1**

(b) 35/2 grams added; accept equivalent exact forms. **A1**

(b) Answer: 35/2 grams

Question 5

$9*x + C$, where C is an arbitrary constant; accept equivalent forms. **B1**

Answer: $9*x + C$

Question 6

Raise each power by 1 and divide each coefficient by the new power. **M1**

$x^5 - (3/2)*x^2 + C$; accept equivalent forms. **A1**

Answer: $x^5 - (3/2)*x^2 + C$

Question 7

Integrate term by term, including the constant term -6. **M1**

$(1/2)*x^4 + (1/3)*x^3 - 6*x + C$; accept equivalent forms. **A1**

Answer: $(1/2)*x^4 + (1/3)*x^3 - 6*x + C$

Question 8

Integrate to $y = x^4 - x^2 + 3*x + C$, allowing a coefficient error. **M1**

Use (1, 6): $6 = 1 - 1 + 3 + C$, giving $C = 3$. Award for substituting the point into their integrated equation and solving for its constant. **M1**

$y = x^4 - x^2 + 3*x + 3$; accept equivalent forms. **A1**

Answer: $y = x^4 - x^2 + 3*x + 3$

Question 9

Integrate to $y = x^3 + 2*x^2 - 5*x + C$, allowing a coefficient error. **M1**

Use (-1, 2): $2 = -1 + 2 + 5 + C$, giving $C = -4$. Award for substituting the point into their integrated equation and solving for its constant. **M1**

$y = x^3 + 2*x^2 - 5*x - 4$; accept equivalent forms. **A1**

Answer: $y = x^3 + 2*x^2 - 5*x - 4$

Question 10

Integrate term by term. An antiderivative is $F(x) = (2/3)*x^3 - (3/2)*x^2 + 4*x$. **M1**

$F(2) = 22/3$ and $F(-1) = -37/6$, using the displayed F. Accept consistent endpoint values from another antiderivative differing by a constant. **A1**

Subtract the lower-limit value from the upper-limit value: $22/3 - (-37/6)$. **M1**

27/2; accept equivalent exact forms. No arbitrary constant remains. **A1**

Answer: 27/2

Question 11

Integrate term by term. An antiderivative is $F(x) = 2*x^3 + x^2 - x$. **M1**

$F(3/2) = 15/2$ and $F(1/2) = 0$, using the displayed F. Accept consistent endpoint values from another antiderivative differing by a constant. **A1**

Subtract the lower-limit value from the upper-limit value: $15/2 - 0$. **M1**

15/2; accept equivalent exact forms. No arbitrary constant remains. **A1**

Answer: 15/2

Question 12

(a) Integrate each term using the power rule. **M1**

(a) $x^3 - x^2 + 4*x + C$, including the arbitrary constant; accept equivalent forms. **A1**

(a) Answer: $x^3 - x^2 + 4*x + C$

(b) Use $[x^3 - x^2 + 4*x]$ from 0 to 2: $(8 - 4 + 8) - 0$. **M1**

(b) 12, with no arbitrary constant; accept equivalent exact forms. **A1**

(b) Answer: 12; no arbitrary constant remains.

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