HIGHER

5.1 Differentiation: Gradients and Turning Points: Test Pack

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 65 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $y = (2x - 5)(x - 1)$
Find dy/dx.

(Total for Question 1 is 3 marks)

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2 A curve has equation $y = x^3 - 6x - 2$.
Find the coordinates of the points on the curve where the gradient is -3.

(Total for Question 2 is 5 marks)

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3 A ball is thrown upwards. Its height h metres above the ground after t seconds is $h = -5t^2 + 60t$.
Find the maximum height reached by the ball.

(Total for Question 3 is 4 marks)

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4 A curve has equation $y = 5x^4 - 3x^3 - 5x - 1$.
Find the gradient of the curve at the point where $x = -2$.

(Total for Question 4 is 2 marks)

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5 The curve C has equation $y = 4x^3 - x^2 + 2x + 4$.
Find an equation of the normal to C at the point where $x = -2$. Give your answer in the form $y = mx + c$.

(Total for Question 5 is 5 marks)

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6 A curve has equation $y = x^3 + 2x + 4$.
Find the coordinates of the points on the curve where the gradient is 14.

(Total for Question 6 is 5 marks)

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7 A farmer has 16 m of fencing. She uses it to make three sides of a rectangular pen against a straight wall, so the fourth side needs no fencing. The two sides perpendicular to the wall each have length x metres.

Find the value of x that gives the maximum area of the pen, and find this maximum area.

(Total for Question 7 is 5 marks)

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8 A curve has equation $y = x^3 + 6x^2 + 2$.

Find the coordinates of the turning points of the curve and determine the nature of each turning point.

(Total for Question 8 is 6 marks)

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9 A curve has equation $y = -x^3 - 9x^2 - 24x$.
Find the values of x for which y is decreasing (the gradient is negative).

(Total for Question 9 is 4 marks)

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10 The curve C has equation $y = -3x^3 + 2x^2 + 6x + 6$.
Find an equation of the tangent to C at the point where $x = 1$. Give your answer in the form $y = mx + c$.

(Total for Question 10 is 4 marks)

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Mark scheme · 5.1 Differentiation: Gradients and Turning Points: Test Pack

Question 1

expands the brackets to get $y = 2x^2 - 7x + 5$	**M1**
differentiates at least two terms of their expansion correctly	**M1**
$dy/dx = 4x - 7$ cao	**A1**

Answer: $dy/dx = 4x - 7$

Question 2

$dy/dx = 3x^2 - 6$	**M1**
sets $dy/dx = -3$ and rearranges to $3x^2 - 3 = 0$	**M1**
solves the quadratic to get $x = -1$ and $x = 1$	**M1**
substitutes both x values into the equation of the curve	**M1**
(-1, 3) and (1, -7) cao	**A1**

Answer: (-1, 3) and (1, -7)

Question 3

$dh/dt = -10t + 60$	**M1**
sets $dh/dt = 0$ to find the time of maximum height	**M1**
$t = 6$ seconds	**A1**
maximum height = 180 m cao	**A1**

Answer: Maximum height = 180 m

Question 4

$dy/dx = 20x^3 - 9x^2 - 5$ found	**M1**
gradient = -201 cao, from substituting $x = -2$	**A1**

Answer: Gradient = -201

Question 5

$dy/dx = 12x^2 - 2x + 2$	**M1**
gradient of the curve = 54 at $x = -2$	**M1**
gradient of the normal = $-1/(54) = -1/54$	**M1**
$y = -36$ at $x = -2$, so the point is (-2, -36)	**M1**
$y = -(1/54)x - 973/27$ cao	**A1**

Answer: $y = -(1/54)x - 973/27$

Question 6

$dy/dx = 3x^2 + 2$	**M1**
sets $dy/dx = 14$ and rearranges to $3x^2 - 12 = 0$	**M1**
solves the quadratic to get $x = -2$ and $x = 2$	**M1**
substitutes both x values into the equation of the curve	**M1**
(-2, -8) and (2, 16) cao	**A1**

Answer: (-2, -8) and (2, 16)

Question 7

side parallel to the wall = $16 - 2x$	**M1**
$A = x(16 - 2x) = -2x^2 + 16x$	**M1**
$dA/dx = -4x + 16 = 0$	**M1**
$x = 4$ cao	**A1**
maximum area = 32 m² cao	**A1**

Answer: $x = 4$, maximum area = 32 m^2

Question 8

$dy/dx = 3x^2 + 12x$	**M1**
sets $dy/dx = 0$ and solves the quadratic	**M1**
$x = -4$ and $x = 0$	**A1**
substitutes both x values into the equation of the curve	**M1**
turning points at (-4, 34) and (0, 2)	**A1**
$d2y/dx2 = 6x + 12$ shows (-4, 34) is a maximum and (0, 2) is a minimum	**B1**

Answer: (-4, 34) is a maximum and (0, 2) is a minimum

Question 9

$dy/dx = -3x^2 - 18x - 24$	**M1**
solves $dy/dx = 0$ to get $x = -4$ and $x = -2$	**M1**
identifies the sign of dy/dx between and outside these roots (a negative x^2 coefficient)	**M1**
$x < -4$ or $x > -2$ cao	**A1**

Answer: $x < -4$ or $x > -2$

Question 10

$dy/dx = -9x^2 + 4x + 6$	**M1**
gradient of the curve = 1 at $x = 1$	**M1**
$y = 11$ at $x = 1$, so the point is (1, 11)	**M1**
$y = x + 10$ cao	**A1**

Answer: $y = x + 10$

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