Revision Library
revisionlibrary.co.uk

HIGHER

IG.M1 Differentiation: Gradients and Turning Points

EDEXCEL 4MA1 · Calculator allowed · about 80 minutes

Total Marks

Name: ________________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 $y = 9$. Write down the value of dy/dx.

(Total for Question 1 is 1 mark)

2 $y = x^5$. Find dy/dx.

(Total for Question 2 is 1 mark)

3 $y = -3x^2$. Find dy/dx.

(Total for Question 3 is 1 mark)

4 $y = 6x^3 - 4x + 1$. Find dy/dx.

(Total for Question 4 is 2 marks)

5 $y = 5x^2 - 7x - 2$. Find dy/dx.

(Total for Question 5 is 2 marks)

6 $y = 2x^3 + 5x^2$. Find d^2y/dx^2, the second derivative of y with respect to x.

(Total for Question 6 is 2 marks)

---

7 Find the gradient of the curve $y = x^3 - 4x$ at the point where $x = 2$.

(Total for Question 7 is 2 marks)

---

8 The curve $y = 2x^2 - 8x + 3$ has gradient 4 at the point P. Find the x-coordinate of P.

(Total for Question 8 is 3 marks)

---

9 Find the coordinates of the point on the curve $y = x^2 - 6x + 5$ at which the gradient is zero.

(Total for Question 9 is 3 marks)

---

10 $y = x^3 - 3x^2 - 9x + 5$

(a) Find dy/dx. **(2)**

....

(b) Find the x-coordinates of the turning points of the curve. **(3)**

....

(Total for Question 10 is 5 marks)

11 This question continues from Question 10, for the curve $y = x^3 - 3x^2 - 9x + 5$, which has turning points at $x = -1$ and $x = 3$.

(a) Find the y-coordinate of each turning point. **(2)**

....

(b) By finding d^2y/dx^2, determine whether each turning point is a maximum or a minimum. **(3)**

(Total for Question 11 is 5 marks)

---

12 A particle moves in a straight line so that its displacement, s metres, from a fixed point O at time t seconds is given by $s = t^3 - 6t^2 + 9t$, for $t \geq 0$.

(a) Find an expression for the velocity, v m/s, of the particle at time t. **(2)**

....

(b) Find the velocity of the particle when $t = 4$. **(2)**

....

(c) Find the values of t at which the particle is instantaneously at rest. **(3)**

....

(Total for Question 12 is 7 marks)

13 For the particle described in Question 12, with displacement $s = t^3 - 6t^2 + 9t$, find the acceleration of the particle when $t = 1$, and state whether the particle is speeding up or slowing down at this instant.

(Total for Question 13 is 3 marks)

---

14 A farmer has 40 m of fencing. She wants to fence off a rectangular area for sheep, using a straight wall as one side of the rectangle, so fencing is only needed for the other three sides. She uses x metres of fencing for each of the two sides perpendicular to the wall, and the rest of the fencing for the side parallel to the wall. Let A m^2 be the area enclosed.

Figure (to be drawn): *A rectangle representing the sheep enclosure. The top side lies along a straight wall (drawn as a thick line, not fenced). The two vertical sides, each labelled x metres, are perpendicular to the wall. The bottom side, parallel to the wall, is labelled (40 - 2x) metres.*

(a) Show that $A = 40x - 2x^2$. **(2)**

(b) Find dA/dx. **(2)**

....

(c) Find the value of x that gives the maximum possible enclosed area, and find this maximum area. **(3)**

....

(d) Explain how you know that this value of x gives a maximum area, not a minimum area. **(1)**

(Total for Question 14 is 8 marks)

15 $y = x^4 - 8x^2 + 3$

(a) Find dy/dx. **(2)**

....

(b) Find the x-coordinates of all the stationary points of the curve. **(4)**

....

(Total for Question 15 is 6 marks)

---

16 Find the value(s) of x for which the tangent to the curve $y = x^3 - 2x^2 + 1$ is parallel to the line $y = 4x - 5$.

(Total for Question 16 is 4 marks)

---

17 Show that the curve $y = x^3 - 3x^2 + 3x + 5$ has exactly one stationary point, and determine its nature.

(Total for Question 17 is 4 marks)

18 The number of bacteria, N, in a laboratory culture, t hours after the start of an experiment, is modelled by $N = 200 + 30t^2 - 2t^3$, for $0 \leq t \leq 10$.

(a) Find dN/dt. **(2)**

....

(b) Find the rate at which the number of bacteria is increasing when $t = 3$. **(2)**

....

(c) Find the value of t, for $0 \leq t \leq 10$, at which the rate of increase of the number of bacteria is at its greatest. **(3)**

....

(Total for Question 18 is 7 marks)

Mark scheme · IG.M1 Differentiation: Gradients and Turning Points

Question 1

- **B1** $dy/dx = 0$ cao
- **Answer: $dy/dx = 0$**

Question 2

- **B1** $dy/dx = 5x^4$ cao
- **Answer: $dy/dx = 5x^4$**

Question 3

- **B1** $dy/dx = -6x$ cao
- **Answer: $dy/dx = -6x$**

Question 4

- **M1** differentiates at least two terms correctly, e.g. $18x^2$ or -4 seen
- **A1** $dy/dx = 18x^2 - 4$ cao (the constant +1 differentiates to 0)
- **Answer: $dy/dx = 18x^2 - 4$**

Question 5

- **M1** differentiates at least two terms correctly, e.g. $10x$ or -7 seen
- **A1** $dy/dx = 10x - 7$ cao
- **Answer: $dy/dx = 10x - 7$**

Question 6

- **M1** finds $dy/dx = 6x^2 + 10x$ (differentiating once)
- **A1** $d^2y/dx^2 = 12x + 10$ cao (differentiating a second time)
- **Answer: $d^2y/dx^2 = 12x + 10$**

Question 7

- **M1** $dy/dx = 3x^2 - 4$ found
- **A1** gradient = 8 cao, from correctly substituting $x = 2$
- **Answer: Gradient = 8**

Question 8

- **M1** $dy/dx = 4x - 8$ found
- **M1** sets $4x - 8 = 4$ and rearranges, e.g. $4x = 12$
- **A1** $x = 3$ cao
- **Answer: $x = 3$**

Question 9

- **M1** $dy/dx = 2x - 6$ found and set equal to 0
- **A1** $x = 3$ cao
- **A1** $y = -4$ cao, so the point is $(3, -4)$, ft from their x
- **Answer: $(3, -4)$**

Question 10

- (a) **M1** differentiates at least two terms correctly
- (a) **A1** $dy/dx = 3x^2 - 6x - 9$ cao
- **(a) Answer: $dy/dx = 3x^2 - 6x - 9$**
- (b) **M1** sets their $dy/dx = 0$ and simplifies to a 3-term quadratic equal to zero, e.g. $x^2 - 2x - 3 = 0$, ft from part (a)
- (b) **M1** factorises or solves the quadratic correctly, e.g. $(x - 3)(x + 1) = 0$
- (b) **A1** $x = 3$ and $x = -1$ cao (both values)
- **(b) Answer: $x = -1$ and $x = 3$**

Question 11

- (a) **M1** substitutes $x = -1$ and $x = 3$ into $y = x^3 - 3x^2 - 9x + 5$
- (a) **A1** $y = 10$ at $x = -1$ and $y = -22$ at $x = 3$, both correct, cao
- **(a) Answer: (-1, 10) and (3, -22)**
- (b) **M1** finds $d^2y/dx^2 = 6x - 6$
- (b) **A1** at $x = -1$, $d^2y/dx^2 = -12 < 0$, so (-1, 10) is a maximum
- (b) **A1** at $x = 3$, $d^2y/dx^2 = 12 > 0$, so (3, -22) is a minimum
- **(b) Answer: (-1, 10) is a maximum; (3, -22) is a minimum**

Question 12

- (a) **M1** differentiates s with respect to t
- (a) **A1** $v = 3t^2 - 12t + 9$ cao
- **(a) Answer: $v = 3t^2 - 12t + 9$**
- (b) **M1** substitutes $t = 4$ into their expression for v, ft from part (a)
- (b) **A1** $v = 9$ m/s cao
- **(b) Answer: v = 9 m/s**
- (c) **M1** sets their $v = 0$, ft from part (a), e.g. $3t^2 - 12t + 9 = 0$, and simplifies to $t^2 - 4t + 3 = 0$
- (c) **M1** factorises or solves the quadratic correctly, e.g. $(t - 1)(t - 3) = 0$
- (c) **A1** $t = 1$ and $t = 3$ cao (both values)
- **(c) Answer: t = 1 second and t = 3 seconds**

Question 13

- **M1** finds acceleration $a = dv/dt = 6t - 12$
- **M1** substitutes $t = 1$ into their expression for a
- **A1** $a = -6$ m/s^2 cao, with a correct statement that the particle is speeding up (accelerating) in the negative direction: velocity at $t = 1$ is 0 and acceleration is negative, so the particle is momentarily at rest and then moves in the negative direction with increasing speed, oe sensible interpretation accepted
- **Answer: $a = -6$ m/s^2; the particle is momentarily at rest and then speeds up in the negative direction**

Question 14

- (a) **M1** correctly finds the length of the side parallel to the wall as (40 - 2x) metres
- (a) **A1** forms A = x(40 - 2x) and expands fully to $A = 40x - 2x^2$, cso
- **(a) Answer: $A = 40x - 2x^2$ (shown)**
- (b) **M1** differentiates A with respect to x
- (b) **A1** dA/dx = 40 - 4x cao
- **(b) Answer: dA/dx = 40 - 4x**
- (c) **M1** sets their dA/dx = 0 and solves, ft from part (b)
- (c) **A1** x = 10 cao
- (c) **A1** maximum area = 200 m^2 cao, from substituting x = 10 into $A = 40x - 2x^2$ (or into x times the parallel side, length 20 m)
- **(c) Answer: x = 10 m, maximum area = 200 m^2**
- (d) **B1** correct justification, e.g. $d^2A/dx^2 = -4$, which is negative, so it is a maximum; oe (e.g. $A = 40x - 2x^2$ is a negative quadratic in x, so its graph is an upside-down parabola with a single maximum turning point)
- **(d) Answer: $d^2A/dx^2 = -4 < 0$, so it is a maximum**

Question 15

- (a) **M1** differentiates at least one term correctly
- (a) **A1** $dy/dx = 4x^3 - 16x$ cao
- **(a) Answer: $dy/dx = 4x^3 - 16x$**
- (b) **M1** sets their dy/dx = 0, ft from part (a)
- (b) **M1** takes out a common factor correctly, e.g. $4x(x^2 - 4) = 0$
- (b) **A1** x = 0 cao
- (b) **A1** x = 2 and x = -2 cao (both values), from correctly factorising $x^2 - 4 = (x - 2)(x + 2)$
- **(b) Answer: x = -2, x = 0, x = 2**

Question 16

- **M1** finds $dy/dx = 3x^2 - 4x$
- **M1** sets their dy/dx equal to 4 (the gradient of the given line) and rearranges to $3x^2 - 4x - 4 = 0$, oe
- **A1** x = 2 cao
- **A1** x = -2/3 oe cao
- **Answer: x = 2 or x = -2/3**

Question 17

- **M1** finds $dy/dx = 3x^2 - 6x + 3$
- **M1** factorises fully, e.g. $dy/dx = 3(x - 1)^2$
- **A1** correct conclusion that $3(x - 1)^2 \geq 0$ for all x, equal to 0 only when x = 1, so there is exactly one stationary point (at x = 1), cso
- **B1** correctly identifies the nature: since dy/dx does not change sign around x = 1 (it is ≥ 0 on both sides), this is a stationary point of inflection, not a maximum or minimum; oe supported by $d^2y/dx^2 = 6x - 6 = 0$ at x = 1
- **Answer: Exactly one stationary point, at x = 1; it is a stationary point of inflection**

Question 18

- (a) **M1** differentiates N with respect to t
- (a) **A1** $dN/dt = 60t - 6t^2$ cao
- **(a) Answer: $dN/dt = 60t - 6t^2$**
- (b) **M1** substitutes t = 3 into their dN/dt, ft from part (a)
- (b) **A1** 126 (bacteria per hour) cao
- **(b) Answer: 126 bacteria per hour**
- (c) **M1** differentiates dN/dt to find $d^2N/dt^2 = 60 - 12t$
- (c) **M1** sets their $d^2N/dt^2 = 0$ and solves
- (c) **A1** t = 5 cao, with $0 \leq 5 \leq 10$ confirmed to be in the given range