HIGHER

IG.M10 Direct and Inverse Proportion With Non-Linear Powers

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer all questions. Full sentences are required in 2 questions (where an explanation is asked). Allow 60 minutes.

1 Given that y is proportional to x^2 for a square tile, and when x = 4 cm then y = 36 (area units), write the equation relating y and x and find the value of k.

(Total for Question 1 is 1 mark)

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2 Given that T is proportional to v^3 for a model where thrust T increases with the cube of speed v, and T = 54 N when v = 3 m/s, find the constant of proportionality k in $T = kv^3$.

(Total for Question 2 is 1 mark)

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3 Given that y is proportional to $1/\sqrt{x}$ ($y = k / x^{1/2}$) for cooling time y depending inversely on the square root of mass x, and y = 10 when x = 4, find k and state the proportionality equation.

(Total for Question 3 is 2 marks)

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4 A circular pond has water depth y that varies directly as the square of the radius r, $y = kr^2$, in an experimental model. Given that y = 18 when r = 3 m, form the equation and hence find y when r = 5 m.

(Total for Question 4 is 2 marks)

5 Given that the cost C of making a decorative square panel varies as the square of its side length s, $C = ks^2$. If a panel of side 1.2 m costs pound 43.20, find k and then find the cost of a panel of side 0.8 m. Give answers to 2 decimal places.

(Total for Question 5 is 2 marks)

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6 Given that y is proportional to x^3, $y = kx^3$, and y = 16 when x = 2, find k and then hence find x when y = 432.

(Total for Question 6 is 3 marks)

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7 Show that if the period T of a simple pendulum is proportional to the square root of its length L, $T = k L^{1/2}$, and T = 1.8 s when L = 0.81 m, then k = 2. Verify the period when L = 2.25 m.

(Total for Question 7 is 3 marks)

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8 The shaking force F on a small structure varies inversely as the square of the distance d from the source, $F = k / d^2$. If F = 0.5 N at d = 2 m, find k and hence find F at d = 5 m, giving your answer to 3 significant figures.

(Total for Question 8 is 3 marks)

9 A physics experiment models intensity I of radiation as $I = k / r^3$, inversely proportional to the cube of distance r. If I = 0.125 units at r = 2 m, find k and hence find r when I = 0.001 units. Give r to 2 decimal places.

(Total for Question 9 is 4 marks)

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10 A recipe says that the volume V of a loaf varies directly as the 3/2 power of the amount of yeast x used, $V = k x^{3/2}$. If using 8 g of yeast gives volume 1440 cm^3, find k and then the volume when 2 g of yeast is used. Give the final volume to the nearest whole number.

(Total for Question 10 is 4 marks)

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11 A thin metal plate has heat loss rate H that varies as the square root of its area A, $H = k A^{1/2}$. An experiment gives H = 12 W when A = 16 cm^2. Find k and then find H when A = 100 cm^2.

(Total for Question 11 is 4 marks)

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12 Show that if the period P of oscillation of a particle varies inversely as the square of amplitude a, $P = k / a^2$, and P = 0.5 s when a = 0.4 m, then $k = 0.5*(0.4)^2 = 0.08$. Hence find P when a = 0.2 m and explain briefly whether P increases or decreases when amplitude halves.

(Total for Question 12 is 4 marks)

13 A scale model ship's resistance R varies as the square of its speed v and inversely as the cube root of its length L, $R = k\ v^2 / L^{1/3}$. In a test tank, R = 0.96 N when v = 3 m/s and L = 0.125 m. Find k and then find R when v = 5 m/s and L = 1.0 m. Give R to 3 significant figures.

(Total for Question 13 is 5 marks)

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14 A charity fundraiser states that the amount raised A varies directly as the square of the number of attendees n, $A = k\ n^2$. At a small event with n = 30 attendees, A = pound 2700. Use this to estimate how many attendees would be needed to raise pound 3840, assuming the same k. Give your answer to the nearest whole person and show your working in words.

(Total for Question 14 is 5 marks)

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15 A lamp's illuminance E varies inversely as the square root of distance d, $E = k / d^{1/2}$. At d = 0.25 m, E = 80 lux. A photographer asks: how far must the lamp be moved so that illuminance is reduced to 20 lux? Solve for the new distance d and give the answer to 2 decimal places.

(Total for Question 15 is 4 marks)

Mark scheme · IG.M10 Direct and Inverse Proportion With Non-Linear Powers

Question 1

- **B1** $y = kx^2$ with $k = 36/16 = 9/4$ or 2.25 stated
- **Answer: $y = (9/4)x^2$ or $y = 2.25x^2$**

Question 2

- **B1** $k = 54/27 = 2$ stated
- **Answer: $k = 2$**

Question 3

- **M1** substitutes values into $y = k/x^{1/2}$: $10 = k/2$ or equivalent
- **A1** $k = 20$ and equation $y = 20 / x^{1/2}$ or $y = 20\ x^{-1/2}$ cao
- **Answer: $k = 20$, so $y = 20 / x^{1/2}$**

Question 4

- **M1** finds k from $18 = k*3^2$, $k = 2$
- **A1** $y = 2*5^2 = 50$ stated
- **Answer: $y = 50$ (depth units)**

Question 5

- **M1** finds $k = 43.20 / (1.2^2) = 43.20 / 1.44 = 30$
- **A1** cost at 0.8 m is $C = 30 * 0.8^2 = 30 * 0.64 = 19.20$, cao
- **Answer: $k = 30$, cost = pound 19.20**

Question 6

- **M1** finds k from $16 = k*(2^3)$ so $k = 16/8 = 2$
- **M1** substitutes k into $432 = 2*x^3$ and rearranges to $x^3 = 216$
- **A1** $x = 6$ cao
- **Answer: $k = 2$ and $x = 6$**

Question 7

- **M1** substitutes $1.8 = k*(0.81)^{1/2}$ and finds $\sqrt{0.81} = 0.9$ so $k = 1.8/0.9 = 2$
- **M1** uses $k = 2$ to find $T = 2*(2.25)^{1/2}$ and evaluates $\sqrt{2.25} = 1.5$
- **A1** $T = 2*1.5 = 3.0$ s cao
- **Answer: $k = 2$ and $T = 3.0$ s when $L = 2.25$ m**

Question 8

- **M1** finds k from $0.5 = k/2^2$ so $k = 0.5 * 4 = 2$
- **M1** substitutes k to get $F = 2 / 5^2 = 2 / 25 = 0.08$
- **A1** $F = 0.0800...$ so 0.08 N or 0.080 to 3 s.f., accept 0.080 N cao
- **Answer: $k = 2$, $F = 0.0800...$ so 0.080 N (3 s.f.)**

Question 9

- **M1** finds k from $0.125 = k / 2^3$ so $k = 0.125 * 8 = 1$
- **M1** uses $k = 1$ and sets $0.001 = 1 / r^3$ to get $r^3 = 1000$
- **M1** takes cube root to find $r = 10$, or evaluates $r = 1000^{1/3}$
- **A1** $r = 10.00$ m cao
- **Answer: $k = 1$ and $r = 10.00$ m**

Question 10

- **M1** substitutes $1440 = k*(8)^{3/2}$. Recognises $8^{1/2} = 2*\sqrt{2}$ or better to compute $8^{3/2} = (\sqrt{8})^3 = (2.828427...)^3 = 22.627...$ or $8^{3/2} = (8^1)*(8^1/2) = 8*2.828...$
- **M1** calculates $k = 1440 / 8^{3/2} = 1440 / 22.627416997$ = approx 63.64, or uses exact form $k = 1440 / (8*\sqrt{8}) = 45*\sqrt{2}$
- **M1** uses k to find V at x=2: $V = k*(2)^{3/2}$ and evaluates
- **A1** V = 180 cm^3 (nearest whole number) cao
- **Answer: k approx 63.64 (exact $45*\sqrt{2}$), V at 2 g = 180 cm^3**

Question 11

- **M1** finds k from $12 = k*(16)^{1/2}$ so $k = 12/4 = 3$
- **M1** uses k to get $H = 3*(100)^{1/2} = 3*10$
- **A1** H = 30 W cao
- **B1** units W stated with final numeric answer
- **Answer: k = 3, H = 30 W**

Question 12

- **M1** substitutes to find k: $0.5 = k / 0.4^2$ so $k = 0.5 * 0.16 = 0.08$
- **M1** uses k to find P at a = 0.2: $P = 0.08 / 0.2^2 = 0.08 / 0.04 = 2$
- **A1** P = 2.0 s cao
- **B1** brief explanation: when amplitude halves, denominator a^2 becomes one quarter, so P becomes four times larger, so P increases, oe
- **Answer: k = 0.08, P = 2.0 s; P increases (becomes 4 times larger) when amplitude halves**

Question 13

- **M1** substitutes $0.96 = k*(3^2) / (0.125)^{1/3}$ and recognises $3^2 = 9$ and cube root 0.125 = 0.5
- **M1** computes $k = 0.96 * 0.5 / 9 = (0.48)/9 = 0.053333...$ or 0.0533333
- **M1** substitutes k into $R = k*v^2 / L^{1/3}$ with v=5 and L=1.0 giving $R = k*25 / 1 = 25k$
- **M1** calculates $R = 25 * 0.053333... = 1.333333...$
- **A1** R = 1.33 N to 3 s.f. cao
- **Answer: k = 0.053333... (16/300), R = 1.33 N (3 s.f.)**

Question 14

- **M1** finds k from $2700 = k*30^2$ so $k = 2700/900 = 3$
- **M1** sets $3840 = 3*n^2$ and rearranges to $n^2 = 1280$
- **M1** takes square root to get $n = \sqrt{1280}$ and evaluates
- **A1** n approx 35.777... and rounded to nearest whole person gives 36 cao
- **B1** brief wording: states using the same k, $n = \sqrt{3840/3}$ approx 35.78, so 36 attendees needed, oe
- **Answer: n approx 35.78, so 36 attendees (nearest whole person)**

Question 15

- **M1** finds k from $80 = k / 0.25^{1/2}$; $\sqrt{0.25}$ =0.5 so $k = 80*0.5 = 40$
- **M1** sets $20 = 40 / d^{1/2}$ and rearranges to $d^{1/2} = 40/20 = 2$
- **M1** squares to get d = 4 and evaluates
- **A1** d = 4.00 m cao
- **Answer: d = 4.00 m**