HIGHER

2.6 Transformations of Graphs of Functions: 2-hour Pack

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The curve y = f(x) has a minimum point at (3, 6).
Write down the coordinates of the minimum point on the curve y = 2f(x) - 1.

(Total for Question 1 is 3 marks)

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2 The graph of y = f(x) is transformed to give the graph of y = f(x) - 2.
Given that $f(x) = -x^2 + 6x - 3$, find the equation of the transformed graph in the form y = ... ,
with the right-hand side expanded and simplified.

(Total for Question 2 is 3 marks)

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3 The graph of $y = x^2$ is translated by the vector [3; 4].
Find the equation of the new graph. Give your answer in the form y = ... , expanded and simplified.

(Total for Question 3 is 3 marks)

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4 The graph of $y = \cos(2(x - 45))$ is drawn for $x \geq 0$ (x in degrees).
Find the coordinates of the first maximum point of the graph for $x > 0$.

(Total for Question 4 is 4 marks)

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5 The graph of $y = f(x)$ is transformed to give the graph of $y = f(x + 2)$.
Describe fully the single transformation that does this.

(Total for Question 5 is 2 marks)

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6 The curve $y = f(x)$ crosses the y-axis at (0, 7).
Write down the coordinates of the image of the point (0, 7) on the curve $y = f(x - 1) + 3$.

(Total for Question 6 is 3 marks)

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7 The graph of $y = x^2 - 4$ is translated by the vector [3; -5].
Find the equation of the new graph. Give your answer in the form $y = ...$, expanded and simplified.

(Total for Question 7 is 3 marks)

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8 The graph of $y = 3\sin(2x) + 2$ is drawn for $0 \leq x \leq 360$ (x in degrees).
Write down (a) the maximum value of y, (b) the minimum value of y, (c) the period of the graph in degrees.

(Total for Question 8 is 3 marks)

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9 The graph of $y = 4\sin(4(x - 45)) + 1$ is drawn for $x \geq 0$ (x in degrees).
Find the coordinates of the first maximum point of the graph for $x > 0$.

(Total for Question 9 is 4 marks)

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10 The graph of $y = f(x)$ is transformed to give the graph of $y = 3f(x) + 1$.
Describe fully the two transformations that do this, in the order they are applied.

(Total for Question 10 is 4 marks)

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11 The curve $y = f(x)$ has a maximum point at $(1, 6)$.
Write down the coordinates of the maximum point on the curve $y = f(x - 1) + 3$.

(Total for Question 11 is 3 marks)

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12 The graph of $y = f(x)$ is transformed to give the graph of $y = -2f(x) - 1$.
Given that $f(x) = -x^2 + 3x - 4$, find the equation of the transformed graph in the form $y = ...$, with the right-hand side expanded and simplified.

(Total for Question 12 is 3 marks)

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13 The graph of $y = -x^2$ is translated by the vector $[-1; -2]$.
Find the equation of the new graph. Give your answer in the form $y = ...$, expanded and simplified.

(Total for Question 13 is 3 marks)

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14 The graph of $y = 2\cos(4(x - 45)) + 1$ is drawn for $x \geq 0$ (x in degrees).
Find the coordinates of the first maximum point of the graph for $x > 0$.

(Total for Question 14 is 4 marks)

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15 The graph of $y = f(x)$ is transformed to give the graph of $y = f(x + 2) + 4$.
Describe fully the single transformation that does this.

(Total for Question 15 is 2 marks)

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16 The curve $y = f(x)$ has a minimum point at $(0, -4)$.
Write down the coordinates of the maximum point on the curve $y = -f((1/2)x + 2) + 1$.

(Total for Question 16 is 3 marks)

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17 The graph of $y = f(x)$ is transformed to give the graph of $y = f(x - 3) - 2$.
Given that $f(x) = 3x^2 - 4x - 5$, find the equation of the transformed graph in the form $y = ...$, with the right-hand side expanded and simplified.

(Total for Question 17 is 3 marks)

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18 The graph of $y = x^3 - x$ is translated by the vector $[3; 4]$.
Find the equation of the new graph. Give your answer in the form $y = ...$, expanded and simplified.

(Total for Question 18 is 3 marks)

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19 The graph of $y = \sin(2x) - 2$ is drawn for $0 \leq x \leq 360$ (x in degrees).
Write down (a) the maximum value of y, (b) the minimum value of y, (c) the period of the graph in degrees.

(Total for Question 19 is 3 marks)

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20 The graph of $y = 4\sin(4(x - 120))$ is drawn for $x \geq 0$ (x in degrees).
Find the coordinates of the first maximum point of the graph for $x > 0$.

(Total for Question 20 is 4 marks)

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21 The graph of $y = 4\cos(3x) + 2$ is drawn for $0 \leq x \leq 360$ (x in degrees).
Write down (a) the maximum value of y, (b) the minimum value of y, (c) the period of the graph in degrees.

(Total for Question 21 is 3 marks)

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22 The graph of $y = f(x)$ is transformed to give the graph of $y = -f(x)$.
Describe fully the single transformation that does this.

(Total for Question 22 is 2 marks)

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23 The point P(8, 4) lies on the curve $y = f(x)$.
Write down the coordinates of the image of P on the curve $y = -f((1/2)x - 2) + 3$.

(Total for Question 23 is 3 marks)

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24 The graph of $y = f(x)$ is transformed to give the graph of $y = -f(x + 2)$.
Given that $f(x) = x^3 - 2x^2 - 1$, find the equation of the transformed graph in the form $y = ...$, with the right-hand side expanded and simplified.

(Total for Question 24 is 3 marks)

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25 The graph of $y = 2x^2$ is translated by the vector $[-1; 2]$.
Find the equation of the new graph. Give your answer in the form $y = ...$, expanded and simplified.

(Total for Question 25 is 3 marks)

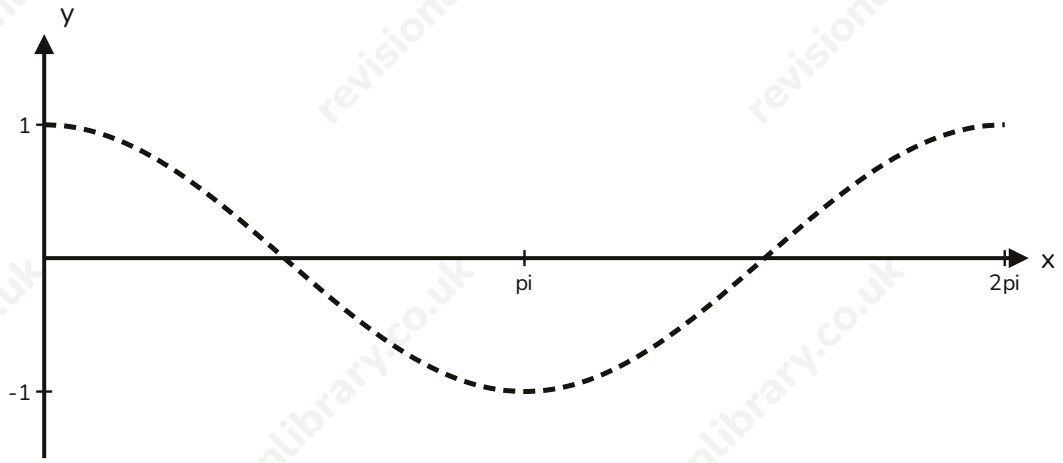
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26 The graph of $y = 2\sin(4x) - 1$ is drawn for $0 \leq x \leq 360$ (x in degrees).
Write down (a) the maximum value of y, (b) the minimum value of y, (c) the period of the graph in degrees.

(Total for Question 26 is 3 marks)

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27 Sketch $y = \cos(2x)$ for $0 \leq x \leq 2\text{pi}$ on the same axes as $y = \cos x$ (faint). State the x-coordinate of the first positive x where $\cos(2x) = 1$.



(Total for Question 27 is 3 marks)

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Mark scheme · 2.6 Transformations of Graphs of Functions: 2-hour Pack

Question 1

identifies the effect of each part of the transformation (the inside of the bracket changes x, the outside changes y)	**M1**
x-coordinate = 3	**B1**
y-coordinate = 11	**B1**

Answer: (3, 11)

Question 2

applies the change outside the function to f(x)	**M1**
expands brackets and collects like terms (applying any multiplier and constant)	**M1**
$y = -x^2 + 6x - 5$ cao	**A1**

Answer: $y = -x^2 + 6x - 5$

Question 3

a translation of [3; 4] replaces x by (x - (3)) and adds 4 to y	**M1**
writes y = f(x - (3)) + 4 with $f(x) = x^2$ and expands	**M1**
$y = x^2 - 6x + 13$ cao	**A1**

Answer: $y = x^2 - 6x + 13$

Question 4

maximum of cos occurs when its argument is 0 degrees (plus multiples of 360)	**M1**
solves 2(x - 45) = 0 + 360n for the smallest positive x	**M1**
x = 45	**A1**
y = 1, so the point is (45, 1)	**A1**

Answer: (45, 1)

Question 5

a change inside the brackets translates the graph horizontally, in the opposite direction to the sign	**M1**
Translation by the vector [-2; 0]	**A1**

Answer: Translation by the vector [-2; 0]

Question 6

identifies the effect of each part of the transformation (the inside of the bracket changes x, the outside changes y)	**M1**
x-coordinate = 1	**B1**
y-coordinate = 10	**B1**

Answer: (1, 10)

Question 7

a translation of [3; -5] replaces x by (x - (3)) and adds -5 to y	**M1**
writes y = f(x - (3)) - 5 with $f(x) = x^2 - 4$ and expands	**M1**
$y = x^2 - 6x$ cao	**A1**

Answer: $y = x^2 - 6x$

Question 8

maximum value = 3 + (2) = 5	**B1**
minimum value = -3 + (2) = -1	**B1**
period = 360/2 = 180 degrees	**B1**

Answer: (a) 5; (b) -1; (c) 180 degrees

Question 9

maximum of sin occurs when its argument is 90 degrees (plus multiples of 360)	**M1**
solves 4(x - 45) = 90 + 360n for the smallest positive x	**M1**
x = 135/2	**A1**
y = 5, so the point is (135/2, 5)	**A1**

Answer: (135/2, 5)

Question 10

the multiplication is applied first, then the constant is added	**M1**
Stretch parallel to the y-axis with scale factor 3 identified	**A1**
second transformation identified correctly and in the correct order	**M1**
Stretch parallel to the y-axis with scale factor 3, then translation by the vector [0; 1] Also accept: translate up 1/3, then stretch vertically by factor 3 (3(f(x)+1/3) = 3f(x)+1). Award all 4 marks: M1 for the initial translation, A1 for its correct direction and distance, M1 for applying the stretch second, and A1 for its correct scale factor.	**A1**

Answer: Stretch parallel to the y-axis with scale factor 3, then translation by the vector [0; 1]

Question 11

identifies the effect of each part of the transformation (the inside of the bracket changes x, the outside changes y)	**M1**
x-coordinate = 2	**B1**
y-coordinate = 9	**B1**

Answer: (2, 9)

Question 12

applies the change outside the function to f(x)	**M1**
expands brackets and collects like terms (applying any multiplier and constant)	**M1**
$y = 2x^2 - 6x + 7$ cao	**A1**

Answer: $y = 2x^2 - 6x + 7$

Question 13

a translation of [-1; -2] replaces x by (x - (-1)) and adds -2 to y	**M1**
writes y = f(x - (-1)) - 2 with $f(x) = -x^2$ and expands	**M1**
$y = -x^2 - 2x - 3$ cao	**A1**

Answer: $y = -x^2 - 2x - 3$

Question 14

maximum of cos occurs when its argument is 0 degrees (plus multiples of 360)	**M1**
solves 4(x - 45) = 0 + 360n for the smallest positive x	**M1**
x = 45	**A1**
y = 3, so the point is (45, 3)	**A1**

Answer: (45, 3)

Question 15

the bracket gives the horizontal translation and the constant gives the vertical translation	**M1**
Translation by the vector [-2; 4]	**A1**

Answer: Translation by the vector [-2; 4]

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Question 16

identifies the effect of each part of the transformation (the inside of the bracket changes x, the outside changes y)	**M1**
x-coordinate = -4	**B1**
y-coordinate = 5	**B1**

Answer: (-4, 5)

Question 17

replaces x by x - 3 in f(x)	**M1**
expands brackets and collects like terms (applying any multiplier and constant)	**M1**
$y = 3x^2 - 22x + 32$ cao	**A1**

Answer: $y = 3x^2 - 22x + 32$

Question 18

a translation of [3; 4] replaces x by (x - (3)) and adds 4 to y	**M1**
writes $y = f(x - (3)) + 4$ with $f(x) = x^3 - x$ and expands	**M1**
$y = x^3 - 9x^2 + 26x - 20$ cao	**A1**

Answer: $y = x^3 - 9x^2 + 26x - 20$

Question 19

maximum value = 1 + (-2) = -1	**B1**
minimum value = -1 + (-2) = -3	**B1**
period = 360/2 = 180 degrees	**B1**

Answer: (a) -1; (b) -3; (c) 180 degrees

Question 20

maximum of sin occurs when its argument is 90 degrees (plus multiples of 360)	**M1**
solves 4(x - 120) = 90 + 360n for the smallest positive x	**M1**
x = 105/2	**A1**
y = 4, so the point is (105/2, 4)	**A1**

Answer: (105/2, 4)

Question 21

maximum value = 4 + (2) = 6	**B1**
minimum value = -4 + (2) = -2	**B1**
period = 360/3 = 120 degrees	**B1**

Answer: (a) 6; (b) -2; (c) 120 degrees

Question 22

replacing f(x) by -f(x) changes the sign of every y value	**M1**
Reflection in the x-axis	**A1**

Answer: Reflection in the x-axis

Question 23

identifies the effect of each part of the transformation (the inside of the bracket changes x, the outside changes y)	**M1**
x-coordinate = 20	**B1**
y-coordinate = -1	**B1**

Answer: (20, -1)

Question 24

replaces x by x + 2 in f(x)	**M1**
expands brackets and collects like terms (applying any multiplier and constant)	**M1**
$y = -x^3 - 4x^2 - 4x + 1$ cao	**A1**

Answer: $y = -x^3 - 4x^2 - 4x + 1$

Question 25

a translation of [-1; 2] replaces x by (x - (-1)) and adds 2 to y	**M1**
writes $y = f(x - (-1)) + 2$ with $f(x) = 2x^2$ and expands	**M1**
$y = 2x^2 + 4x + 4$ cao	**A1**

Answer: $y = 2x^2 + 4x + 4$

Question 26

maximum value = 2 + (-1) = 1	**B1**
minimum value = -2 + (-1) = -3	**B1**
period = 360/4 = 90 degrees	**B1**

Answer: (a) 1; (b) -3; (c) 90 degrees

Question 27

sketch shows horizontal compression with period π and correct amplitude 1	**M1**
shows cos(2x) = 1 at x = 0 and at least one further positive x correctly placed	**M1**
first positive x where cos(2x)=1 given as x = π	**A1**

Answer: First positive x where cos(2x) = 1 is x = π

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