HIGHER

IG.M11 Transformations of Graphs of Functions

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 40 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The graph of y = f(x) is a parabola with vertex at (2, -3) and passes through (0, 1). Sketch the graph of y = f(x) + 4 on the same axes and label the new vertex coordinate.

Figure (to be drawn): A set of coordinate axes is provided for sketching. Labelled point on original parabola: vertex (2, -3) and another point (0, 1). Student should sketch transformed parabola.

(Total for Question 1 is 1 mark)

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2 The graph of y = f(x) has a maximum at (-1, 2). Write down the coordinates of the minimum of y = -f(x).

(Total for Question 2 is 1 mark)

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3 A cubic curve y = f(x) has x-intercepts at x = -2, 0 and 3. Sketch the graph of y = f(x - 1) and state the new x-intercepts.

Figure (to be drawn): Coordinate axes with the original cubic sketched crossing x-axis at -2, 0 and 3. The cubic shape should be evident (end behaviour not required in detail).

(Total for Question 3 is 2 marks)

4 The graph of y = f(x) is shown and has a labelled point A at (3, 2). Describe the single transformation that maps the graph of y = f(x) onto the graph of y = 2 f(x). Also write down the coordinates of the image of A.

Figure (to be drawn): A sketch of a curve with a marked point A at (3, 2).

(Total for Question 4 is 2 marks)

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5 Graph of y = f(x) has a minimum at (0, -5). State the coordinates of the minimum of y = f(2x).

(Total for Question 5 is 1 mark)

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6 The graph of y = f(x) has an x-intercept at (4, 0). State the x-intercept of y = f(x + 3).

(Total for Question 6 is 1 mark)

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7 A trig graph y = f(x) = sin x has a maximum at P at x = π/2 with coordinates (π/2, 1). Sketch y = sin(x/2) on the same axes for $0 \leq x \leq 4pi$ and write down the x-coordinate of the first maximum to the right of x = 0.

Figure (to be drawn): Axes from x = 0 to x = 4pi labelled, with original sin x curve faintly shown and point P at (π/2, 1).

(Total for Question 7 is 3 marks)

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8 The graph of y = f(x) has a point B at (-2, 3). Describe the single transformation that maps y = f(x) onto y = f(x) - 2, and write down the coordinates of the image of B.

(Total for Question 8 is 2 marks)

9 Point C is on the graph of y = f(x) at C = (1, -4). State the image of C under the transformation y = -f(x) + 1. Give a brief reason.

(Total for Question 9 is 2 marks)

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10 The graph of y = f(x) is a parabola with vertex at (1, 0) and passes through (0, 1). Sketch the graph of y = 2 - f(x) on the same axes and label the new vertex coordinate.

Figure (to be drawn): *Axes with the original parabola faintly shown; original vertex (1, 0) and point (0,1) labelled.*

(Total for Question 10 is 2 marks)

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11 The graph of y = f(x) has a maximum at (-1, 4). State the coordinates of this maximum after the transformation y = f(3x). Explain briefly.

(Total for Question 11 is 2 marks)

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12 Sketch y = cos(2x) for $0 \le x \le 2\pi$ on the same axes as y = cos x (faint). State the x-coordinate of the first positive x where cos(2x) = 1.

Figure (to be drawn): *Axes from 0 to 2pi with faint cos x curve shown. Student to draw cos 2x and label relevant point.*

(Total for Question 12 is 3 marks)

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13 A point D on y = f(x) is at D = (2, -1). Find the image of D under the transformation y = 0.5 f(x + 2) - 3. Give a brief explanation of each step.

(Total for Question 13 is 3 marks)

14 The graph of $y = f(x)$ has a labelled point E at $(4, -1)$. State the image of E under the single transformation that maps $y = f(x)$ to $y = 3\,f(x - 2) + 5$. Give a brief reason.

(Total for Question 14 is 3 marks)

Mark scheme · IG.M11 Transformations of Graphs of Functions

Question 1

- **B1** sketch shows parabola shifted vertically up by 4 with correct orientation and position, and new vertex labelled (2, 1) cao
- **Answer: New vertex (2, 1)**

Question 2

- **B1** minimum is at (-1, -2) cao
- **Answer: (-1, -2)**

Question 3

- **M1** sketch shows the cubic shifted right by 1, preserving shape and relative turning points
- **A1** new x-intercepts written as -1, 1 and 4 cao
- **Answer: x-intercepts: -1, 1 and 4**

Question 4

- **M1** correct description: stretch vertically by a factor of 2 with respect to the x-axis, oe
- **A1** coordinates of image given as (3, 4) cao
- **Answer: Stretch vertically by factor 2 in the y-direction; A maps to (3, 4)**

Question 5

- **B1** minimum remains at x = 0 with same y, so (0, -5) cao
- **Answer: (0, -5)**

Question 6

- **B1** x-intercept is at (1, 0) cao
- **Answer: (1, 0)**

Question 7

- **M1** sketch shows horizontal stretch by factor 2 with doubled period, peaks at correct relative positions, and correct amplitude 1
- **M1** student shows at least one maximum at x = π with correct labelling or peak location ft their sketch
- **A1** first maximum to the right of 0 stated as x = π cao
- **Answer: First maximum at x = π**

Question 8

- **M1** description: translate vertically down by 2 units (or shift down 2), oe
- **A1** image coordinates (-2, 1) cao
- **Answer: Translate down 2 units; B maps to (-2, 1)**

Question 9

- **M1** correct image coordinates (1, 5) stated
- **A1** brief reason: reflect in x-axis then translate up 1 so y = -(-4) + 1 = 5, oe
- **Answer: (1, 5)**

Question 10

- **M1** sketch shows reflection in x-axis of original parabola and a translation up by 2 units, preserving shape
- **A1** new vertex labelled at (1, 2) cao
- **Answer: New vertex at (1, 2)**

Question 11

- **M1** recognition that x-coordinate changes by factor 1/3 for points not at x = 0, or that maxima x coordinate becomes -1/3
- **A1** coordinates (-1/3, 4) cao
- **Answer: (-1/3, 4)**

Question 12

- **M1** sketch shows horizontal compression with period π and correct amplitude 1
- **M1** shows cos(2x) = 1 at x = 0 and at least one further positive x correctly placed
- **A1** first positive x where cos(2x)=1 given as x = π
- **Answer: First positive x where cos(2x) = 1 is x = π**

Question 13

- **M1** correctly applies horizontal translation: x comes from solving x + 2 = 2 so original x for image is 0, or explains point mapping: point (2, -1) on f corresponds to x = 2, so for f(x + 2) the same y occurs at x = 0
- **M1** applies vertical scale and translation: y = 0.5*(-1) - 3 = -0.5 - 3
- **A1** final coordinates (0, -3.5) cao
- **Answer: (0, -3.5)**

Question 14

- **M1** recognises horizontal shift right by 2: x = 4 becomes x = 6
- **M1** applies vertical stretch factor 3 then translation up 5: y = -1 -> 3 * (-1) + 5 = 2
- **A1** final image (6, 2) cao
- **Answer: (6, 2)**