HIGHER

2.7 Functions: Domain and Range Restrictions: 2-hour Pack

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $f(x) = x^2 + 6x + 10$, for $x \leq k$.

Find the largest value of k for which f has an inverse.

(Total for Question 1 is 3 marks)

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2 $f(x) = 1/x$

$g(x) = 2x - 6$

Find fg(x) and state the largest possible domain of fg.

(Total for Question 2 is 3 marks)

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3 $f(x) = 2/(x + 1)$

State the value of x that must be excluded from the domain of f.

(Total for Question 3 is 2 marks)

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4 $f(x) = \sqrt{-x + 7}$

Find the largest possible domain of f, giving your answer as an inequality for x.

(Total for Question 4 is 2 marks)

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5 $g(x) = (x - 4)^2 + 3$, for $x \geq 4$.

(a) Find $g^{-1}(x)$.

(b) State the domain of g^{-1}.

(Total for Question 5 is 5 marks)

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6 $h(x) = x^2 - 8x + 19$, for $3 \leq x \leq 5$.

Find the range of h.

(Total for Question 6 is 4 marks)

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7 $f(x) = -3/(x + 1) + 4$

(a) State the largest possible domain of f.

(b) State the range of f.

(Total for Question 7 is 3 marks)

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8 $g(x) = x^2 + 8x + 20$, for $x \leq k$.

Find the largest value of k for which g has an inverse.

(Total for Question 8 is 3 marks)

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9 $g(x) = \sqrt{x}$

$h(x) = x - 6$

Find gh(x) and state the largest possible domain of gh.

(Total for Question 9 is 3 marks)

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10 $h(x) = \sqrt{2x + 8}$

Find the largest possible domain of h, giving your answer as an inequality for x.

(Total for Question 10 is 2 marks)

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11 $g(x) = (x + 2)^2 - 1$, for $x \geq -2$.

(a) Find $g^{-1}(x)$.

(b) State the domain of g^{-1}.

(Total for Question 11 is 5 marks)

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12 $g(x) = 1/(x^2 - 7x + 10)$

Find the values of x that must be excluded from the domain of g.

(Total for Question 12 is 3 marks)

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13 $f(x) = -2x^2 - 6$, for $-2 \leq x \leq 2$.

Find the range of f.

(Total for Question 13 is 4 marks)

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14 The function $r(x) = \sqrt{4 - x}$. State the domain in inequality form and give one example value of x inside the domain and one outside.

(Total for Question 14 is 2 marks)

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15 $h(x) = x^2 + 1$, for all real numbers x.

Find the range of h.

(Total for Question 15 is 2 marks)

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16 $f(x) = 6/(x - 4) + 2$

(a) State the largest possible domain of f.

(b) State the range of f.

(Total for Question 16 is 3 marks)

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17 $f(x) = x^2 + 12x + 29$, for $x \geq k$.

Find the smallest value of k for which f has an inverse.

(Total for Question 17 is 3 marks)

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18 $f(x) = \sqrt{x}$

$g(x) = x - 6$

Find fg(x) and state the largest possible domain of fg.

(Total for Question 18 is 3 marks)

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19 $g(x) = 5/(x^2 + 6x + 5)$

Find the values of x that must be excluded from the domain of g.

(Total for Question 19 is 3 marks)

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20 $f(x) = -x^2 + 4x - 10$, for $-2 \leq x \leq 4$.
Find the range of f.

(Total for Question 20 is 4 marks)

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21 $g(x) = 4/(x + 1) - 1$
(a) State the largest possible domain of g.
(b) State the range of g.

(Total for Question 21 is 3 marks)

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22 $g(x) = x^2 - 10x + 21$, for $x \geq k$.
Find the smallest value of k for which g has an inverse.

(Total for Question 22 is 3 marks)

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23 $f(x) = 1/x$
$g(x) = -x - 4$
Find fg(x) and state the largest possible domain of fg.

(Total for Question 23 is 3 marks)

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24 $f(x) = 1/(x^2 - 3x - 10)$

Find the values of x that must be excluded from the domain of f.

(Total for Question 24 is 3 marks)

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25 $h(x) = \sqrt{2x - 9}$

Find the largest possible domain of h, giving your answer as an inequality for x.

(Total for Question 25 is 2 marks)

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26 $f(x) = 1 - x^2$, for all real numbers x.

Find the range of f.

(Total for Question 26 is 2 marks)

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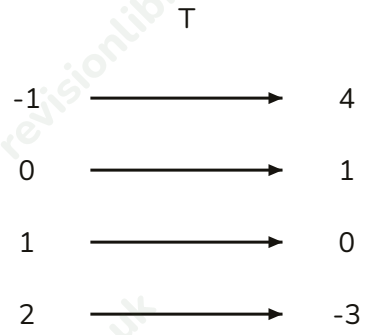
27 $g(x) = x^2 - 6x + 11$, for $x \leq k$.

Find the largest value of k for which g has an inverse.

(Total for Question 27 is 3 marks)

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28 A mapping diagram shows function T: { -1, 0, 1, 2 } -> R with arrows T(-1) = 4, T(0) = 1, T(1) = 0, T(2) = -3. State the domain and range of T using set notation.



(Total for Question 28 is 3 marks)

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Mark scheme · 2.7 Functions: Domain and Range Restrictions: 2-hour Pack

Question 1

completes the square: $f(x) = (x + 3)^2 + 1$	**M1**
turning point at $x = -3$	**A1**
$k = -3$ (the function is one-to-one on one side of its turning point)	**A1**

Answer: $k = -3$

Question 2

$fg(x) = 1/(2x - 6)$	**M1**
requires $2x - 6 \neq 0$	**M1**
$fg(x) = 1/(2x - 6)$, domain $x \neq 3$	**A1**

Answer: $fg(x) = 1/(2x - 6)$, domain $x \neq 3$

Question 3

the denominator cannot be zero: $x + 1 = 0$	**M1**
$x = -1$ is excluded, domain $x \neq -1$	**A1**

Answer: $x \neq -1$

Question 4

the expression under the square root must be at least 0: $-x + 7 \geq 0$	**M1**
$x \leq 7$ (the inequality reverses on dividing by a negative number)	**A1**

Answer: $x \leq 7$

Question 5

$y = (x - 4)^2 + (3)$ and rearranges: $(x - 4)^2 = y - (3)$	**M1**
takes the positive square root because $x \geq 4$	**M1**
$g^{-1}(x) = \sqrt{x - 3} + 4$	**A1**
domain of g^{-1} is the range of g	**B1**
$x \geq 3$	**B1**

Answer: (a) $g^{-1}(x) = \sqrt{x - 3} + 4$; (b) $x \geq 3$

Question 6

completes the square or differentiates to find the turning point at $x = 4$ (value 3)	**M1**
evaluates the end points: $h(3) = 4$, $h(5) = 4$	**M1**
decides whether the turning point $x = 4$ lies in the domain $3 \leq x \leq 5$: it does	**M1**
$3 \leq h(x) \leq 4$	**A1**

Answer: $3 \leq h(x) \leq 4$

Question 7

$x \neq -1$ since the denominator would be 0	**B1**
$-3/(x + 1)$ can take every value except 0, so f(x) can never equal 4	**M1**
range: $f(x) \neq 4$	**A1**

Answer: (a) $x \neq -1$; (b) $f(x) \neq 4$

Question 8

completes the square: $g(x) = (x + 4)^2 + 4$	**M1**
turning point at $x = -4$	**A1**
$k = -4$ (the function is one-to-one on one side of its turning point)	**A1**

Answer: $k = -4$

Question 9

$gh(x) = \sqrt{x - 6}$	**M1**
requires $x - 6 \geq 0$	**M1**
$gh(x) = \sqrt{x - 6}$, domain $x \geq 6$	**A1**

Answer: $gh(x) = \sqrt{x - 6}$, domain $x \geq 6$

Question 10

the expression under the square root must be at least 0: $2x + 8 \geq 0$	**M1**
$x \geq -4$	**A1**

Answer: $x \geq -4$

Question 11

$y = (x + 2)^2 + (-1)$ and rearranges: $(x + 2)^2 = y - (-1)$	**M1**
takes the positive square root because $x \geq -2$	**M1**
$g^{-1}(x) = \sqrt{x + 1} - 2$	**A1**
domain of g^{-1} is the range of g	**B1**
$x \geq -1$	**B1**

Answer: (a) $g^{-1}(x) = \sqrt{x + 1} - 2$; (b) $x \geq -1$

Question 12

sets $x^2 - 7x + 10 = 0$	**M1**
factorises or uses the quadratic formula	**M1**
$x \neq 2$ and $x \neq 5$	**A1**

Answer: $x \neq 2$ and $x \neq 5$

Question 13

completes the square or differentiates to find the turning point at $x = 0$ (value -6)	**M1**
evaluates the end points: $f(-2) = -14$, $f(2) = -14$	**M1**
decides whether the turning point $x = 0$ lies in the domain $-2 \leq x \leq 2$: it does	**M1**
$-14 \leq f(x) \leq -6$	**A1**

Answer: $-14 \leq f(x) \leq -6$

Question 14

states domain $4 - x \geq 0$, leading to $x \leq 4$	**M1**
gives correct examples, e.g. $x = 0$ inside and $x = 5$ outside, or similar, cao	**A1**

Answer: Domain $x \leq 4$; example inside $x = 0$, outside $x = 5$

Question 15

$x^2 \geq 0$ for every real x, with the minimum 0 at $x = 0$	**M1**
$h(x) \geq 1$	**A1**

Answer: $h(x) \geq 1$

Question 16

$x \neq 4$ since the denominator would be 0	**B1**
$6/(x - 4)$ can take every value except 0, so f(x) can never equal 2	**M1**
range: $f(x) \neq 2$	**A1**

Answer: (a) $x \neq 4$; (b) $f(x) \neq 2$

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Question 17

completes the square: $f(x) = (x + 6)^2 - 7$	**M1**
turning point at $x = -6$	**A1**
$k = -6$ (the function is one-to-one on one side of its turning point)	**A1**

Answer: k = -6

Question 18

$fg(x) = \sqrt{x - 6}$	**M1**
requires $x - 6 \geq 0$	**M1**
$fg(x) = \sqrt{x - 6}$, domain $x \geq 6$	**A1**

Answer: $fg(x) = \sqrt{x - 6}$, domain $x \geq 6$

Question 19

sets $x^2 + 6x + 5 = 0$	**M1**
factorises or uses the quadratic formula	**M1**
$x \neq -5$ and $x \neq -1$	**A1**

Answer: x ≠ -5 and x ≠ -1

Question 20

completes the square or differentiates to find the turning point at $x = 2$ (value -6)	**M1**
evaluates the end points: $f(-2) = -22$, $f(4) = -10$	**M1**
decides whether the turning point $x = 2$ lies in the domain $-2 \leq x \leq 4$: it does	**M1**
$-22 \leq f(x) \leq -6$	**A1**

Answer: -22 ≤ f(x) ≤ -6

Question 21

$x \neq -1$ since the denominator would be 0	**B1**
$4/(x + 1)$ can take every value except 0, so g(x) can never equal -1	**M1**
range: $g(x) \neq -1$	**A1**

Answer: (a) x ≠ -1; (b) g(x) ≠ -1

Question 22

completes the square: $g(x) = (x - 5)^2 - 4$	**M1**
turning point at $x = 5$	**A1**
$k = 5$ (the function is one-to-one on one side of its turning point)	**A1**

Answer: k = 5

Question 23

$fg(x) = 1/(-x - 4)$	**M1**
requires $-x - 4 \neq 0$	**M1**
$fg(x) = 1/(-x - 4)$, domain $x \neq -4$	**A1**

Answer: fg(x) = 1/(-x - 4), domain x ≠ -4

Question 24

sets $x^2 - 3x - 10 = 0$	**M1**
factorises or uses the quadratic formula	**M1**
$x \neq -2$ and $x \neq 5$	**A1**

Answer: x ≠ -2 and x ≠ 5

Question 25

the expression under the square root must be at least 0: $2x - 9 \geq 0$	**M1**
$x \geq 9/2$	**A1**

Answer: x ≥ 9/2

Question 26

$x^2 \geq 0$, so $1 - x^2 \leq 1$	**M1**
$f(x) \leq 1$	**A1**

Answer: f(x) ≤ 1

Question 27

completes the square: $g(x) = (x - 3)^2 + 2$	**M1**
turning point at $x = 3$	**A1**
$k = 3$ (the function is one-to-one on one side of its turning point)	**A1**

Answer: k = 3

Question 28

states domain as { -1, 0, 1, 2 }	**M1**
states range as {4, 1, 0, -3} cao	**A1**
both domain and range given in correct set notation	**B1**

Answer: Domain = {-1, 0, 1, 2}; Range = {4, 1, 0, -3}

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