HIGHER

2.7 Functions: Domain and Range Restrictions: 30-minute Pack

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 30 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $f(x) = \sqrt{-x^2 + 36}$

Find the largest possible domain of f, giving your answer as an inequality for x.

(Total for Question 1 is 3 marks)

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2 $g(x) = -2x^2 + 4x$, for $-2 \leq x \leq 4$.

Find the range of g.

(Total for Question 2 is 4 marks)

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3 $f(x) = x^2 + 3$, for all real numbers x.
Find the range of f.

(Total for Question 3 is 2 marks)

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4 Consider the rational function R(x) = (2x + 1)/(x - 2). (a) State all values excluded from the domain because they make the denominator zero. (b) For the domain x > 2, find the range of R. You should show algebraic reasoning: consider behaviour as x -> 2+ and x -> infinity, and check whether R takes any finite minimum or maximum on the domain x > 2. Give the final range in inequality or interval notation.

(Total for Question 4 is 12 marks)

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Mark scheme · 2.7 Functions: Domain and Range Restrictions: 30-minute Pack

Question 1

requires $-x^2 + 36 \geq 0$, so $x^2 \leq 36$	**M1**
critical values $x = 6$ and $x = -6$	**M1**
$-6 \leq x \leq 6$	**A1**

Answer: $-6 \leq x \leq 6$

Question 2

completes the square or differentiates to find the turning point at $x = 1$ (value 2)	**M1**
evaluates the end points: $g(-2) = -16$, $g(4) = -16$	**M1**
decides whether the turning point $x = 1$ lies in the domain $-2 \leq x \leq 4$: it does	**M1**
$-16 \leq g(x) \leq 2$	**A1**

Answer: $-16 \leq g(x) \leq 2$

Question 3

$x^2 \geq 0$ for every real x, with the minimum 0 at $x = 0$	**M1**
$f(x) \geq 3$	**A1**

Answer: $f(x) \geq 3$

Question 4

states $x = 2$ is excluded from the domain because denominator $x - 2 = 0$	**M1**
rearranges R(x) into form $2 + 5/(x - 2)$ or equivalent algebraic manipulation, showing $R(x) = 2 + 5/(x - 2)$	**M1**
uses form to consider limits: as $x \to 2+$ then $5/(x - 2) \to +\infty$ so $R \to +\infty$, showing no upper bound	**M1**
as $x \to +\infty$, $5/(x - 2) \to 0$ so $R \to 2$, indicating a horizontal asymptote at $y = 2$	**M1**
checks monotonic behaviour on $x > 2$, for example differentiating R or noting $5/(x - 2)$ is positive and decreases on $x > 2$, so R decreases from $+\infty$ toward 2	**M1**
concludes that R(x) takes every value greater than 2 on $x > 2$, so range is (2, infinity) cao	**A1**
correct statement that $y = 2$ is not attained for finite $x > 2$, since $5/(x - 2)$ would have to be 0 which only occurs as $x \to \infty$	**B1**
final presentation of domain exclusion and range for $x > 2$, domain $x \neq 2$ and range (2, infinity), cao	**A1**
alternative valid reasoning accepted: solves $R(x) = y$ for x and shows for $x > 2$ this yields $y > 2$, oe	**M1**
correct algebraic manipulation in alternative route, e.g. solving $y = (2x + 1)/(x - 2)$ leading to $x = (1 + 2y)/(y - 2)$, and concluding $y > 2$ for $x > 2$, cao	**A1**
clear statement of excluded value(s) in part (a) and clear linking words in part (b) showing consideration of endpoints and limits	**B1**
final boxed answer: Domain excludes $x = 2$; Range for $x > 2$ is (2, infinity) cao	**A1**

Answer: Excluded: $x = 2$. For domain $x > 2$, range is (2, infinity).

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