HIGHER

2.7 Functions: Domain and Range Restrictions: Test Pack

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 The function f(x) = 5 has domain all real numbers. State the value of f(2) for this constant function and explain why there is no domain restriction in this context.

(Total for Question 1 is 1 mark)

2 $f(x) = (x - 5)^2 + 5$, for $x \leq 5$.

(a) Find $f^{-1}(x)$.

(b) State the domain of f^{-1}.

(Total for Question 2 is 5 marks)

3 $g(x) = x^2 + 10x + 30$, for $x \geq k$.

Find the smallest value of k for which g has an inverse.

(Total for Question 3 is 3 marks)

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4 $h(x) = 4/(x + 3) - 3$

(a) State the largest possible domain of h.

(b) State the range of h.

(Total for Question 4 is 3 marks)

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5 $f(x) = \sqrt{x - 3}$

Find the largest possible domain of f, giving your answer as an inequality for x.

(Total for Question 5 is 2 marks)

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6 $h(x) = (x - 2)^2 - 2$, for $x \geq 2$.

Find the range of h, and state the value of h(4).

(Total for Question 6 is 3 marks)

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7 $g(x) = (x - 2)^2 - 2$, for $x \geq 2$.

Find the range of g, and state the value of g(4).

(Total for Question 7 is 3 marks)

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8 $f(x) = 7/(3x + 1)$

State the value of x that must be excluded from the domain of f.

(Total for Question 8 is 2 marks)

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9 $h(x) = \sqrt{x + 8}$

Find the largest possible domain of h, giving your answer as an inequality for x.

(Total for Question 9 is 2 marks)

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10 $f(x) = \sqrt{-3x - 2}$

Find the largest possible domain of f, giving your answer as an inequality for x.

(Total for Question 10 is 2 marks)

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11 $h(x) = -3/(x - 1) + 2$

(a) State the largest possible domain of h.

(b) State the range of h.

(Total for Question 11 is 3 marks)

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12 For the function $p(x) = (x^2 - 1)/(x + 1)$, state any value(s) excluded from the domain and give the domain in set notation.

(Total for Question 12 is 2 marks)

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13 Given $s(x) = 2/(x^2 - 9)$, find all x excluded from the domain and give the domain in interval notation.

(Total for Question 13 is 2 marks)

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14 Let $Q(x) = x^2 - 4x + 3$. The domain of Q is restricted to $1 \leq x \leq 4$ for this question. Find the range of Q on this restricted domain by locating its vertex and evaluating Q at the vertex and at the domain endpoints. Give the range in inequality form.

(Total for Question 14 is 8 marks)

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Mark scheme · 2.7 Functions: Domain and Range Restrictions: Test Pack

Question 1

f(2) = 5 cao **B1**

Answer: f(2) = 5

Question 2

$y = (x - 5)^2 + (5)$ and rearranges: $(x - 5)^2 = y - (5)$ **M1**

takes the negative square root because $x \leq 5$ **M1**

$f^{-1}(x) = -\sqrt{x - 5} + 5$ **A1**

domain of f^{-1} is the range of f **B1**

$x \geq 5$ **B1**

Answer: (a) $f^{-1}(x) = -\sqrt{x - 5} + 5$; (b) $x \geq 5$

Question 3

completes the square: $g(x) = (x + 5)^2 + 5$ **M1**

turning point at x = -5 **A1**

k = -5 (the function is one-to-one on one side of its turning point) **A1**

Answer: k = -5

Question 4

$x \neq -3$ since the denominator would be 0 **B1**

4/(x + 3) can take every value except 0, so h(x) can never equal -3 **M1**

range: $h(x) \neq -3$ **A1**

Answer: (a) $x \neq -3$; (b) $h(x) \neq -3$

Question 5

the expression under the square root must be at least 0: $x - 3 \geq 0$ **M1**

$x \geq 3$ **A1**

Answer: $x \geq 3$

Question 6

the squared bracket is at least 0 for $x \geq 2$ **M1**

$h(x) \geq -2$ **A1**

h(4) = 4 - 2 = 2 **B1**

Answer: $h(x) \geq -2$; h(4) = 2

Question 7

the squared bracket is at least 0 for $x \geq 2$ **M1**

$g(x) \geq -2$ **A1**

g(4) = 4 - 2 = 2 **B1**

Answer: $g(x) \geq -2$; g(4) = 2

Question 8

the denominator cannot be zero: 3x + 1 = 0 **M1**

x = -1/3 is excluded, domain $x \neq -1/3$ **A1**

Answer: $x \neq -1/3$

Question 9

the expression under the square root must be at least 0: $x + 8 \geq 0$ **M1**

$x \geq -8$ **A1**

Answer: $x \geq -8$

Question 10

the expression under the square root must be at least 0: $-3x - 2 \geq 0$ **M1**

$x \leq -2/3$ (the inequality reverses on dividing by a negative number) **A1**

Answer: $x \leq -2/3$

Question 11

$x \neq 1$ since the denominator would be 0 **B1**

-3/(x - 1) can take every value except 0, so h(x) can never equal 2 **M1**

range: $h(x) \neq 2$ **A1**

Answer: (a) $x \neq 1$; (b) $h(x) \neq 2$

Question 12

identifies x = -1 as a value making denominator zero **M1**

domain = {x : x in R, $x \neq -1$} or written as (-infinity, -1) union (-1, infinity) cao **A1**

Answer: Excluded x = -1; domain {x : x in R, $x \neq -1$}

Question 13

solves $x^2 - 9 = 0$ to get x = 3 and x = -3 **M1**

domain = (-infinity, -3) union (-3, 3) union (3, infinity) cao **A1**

Answer: Excluded x = -3 and x = 3; domain (-infinity, -3) U (-3, 3) U (3, infinity)

Question 14

completes the square or uses vertex formula to find x-coordinate of vertex $x_v = 2$ **M1**

evaluates Q at the vertex to find Q(2) = -1 **M1**

evaluates Q at the endpoints Q(1) = 0 and Q(4) = 3 **M1**

compares values and identifies minimum and maximum correctly, showing minimum -1 at x = 2 and maximum 3 at x = 4 **M1**

states the range in correct inequality form, $-1 \leq Q(x) \leq 3$ cao **A1**

explicit statement that the vertex lies inside the domain and is used to find the minimum, oe **B1**

correct numerical substitution at endpoints and vertex with no arithmetic error, cao **A1**

final answer presented clearly as the set of possible outputs, $-1 \leq Q(x) \leq 3$ **B1**

Answer: $-1 \leq Q(x) \leq 3$

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