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HIGHER

IG.M14 Functions: Domain and Range Restrictions

EDEXCEL 4MA1 · Calculator allowed · about 50 minutes

Total Marks

Name: ______________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 The function f(x) = 5 has domain all real numbers. State the value of f(2) for this constant function and explain why there is no domain restriction in this context.

(Total for Question 1 is 1 mark)

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2 Consider g(x) = 3/(x - 4). State the single value of x that must be excluded from the domain because it makes the expression undefined.

(Total for Question 2 is 1 mark)

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3 h(x) = $\sqrt{x + 2}$. State the restriction on x so that h(x) is real, and give the domain in inequality form.

(Total for Question 3 is 1 mark)

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4 For the function p(x) = $(x^2 - 1)/(x + 1)$, state any value(s) excluded from the domain and give the domain in set notation.

(Total for Question 4 is 2 marks)

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5 The function r(x) = $\sqrt{4 - x}$. State the domain in inequality form and give one example value of x inside the domain and one outside.

(Total for Question 5 is 2 marks)

6 Given $s(x) = 2/(x^2 - 9)$, find all x excluded from the domain and give the domain in interval notation.

(Total for Question 6 is 2 marks)

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7 A mapping diagram shows function T: { -1, 0, 1, 2 } -> R with arrows T(-1) = 4, T(0) = 1, T(1) = 0, T(2) = -3. State the domain and range of T using set notation.

(Total for Question 7 is 3 marks)

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8 Consider the linear function $L(x) = -2x + 5$ with domain x in [0, 3]. Find the range of L over this restricted domain and give it in inequality form.

(Total for Question 8 is 3 marks)

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9 Let $Q(x) = x^2 - 4x + 3$. The domain of Q is restricted to $1 \le x \le 4$ for this question. Find the range of Q on this restricted domain by locating its vertex and evaluating Q at the vertex and at the domain endpoints. Give the range in inequality form.

(Total for Question 9 is 8 marks)

10 Consider the rational function $R(x) = (2x + 1)/(x - 2)$. (a) State all values excluded from the domain because they make the denominator zero. (b) For the domain $x > 2$, find the range of R. You should show algebraic reasoning: consider behaviour as $x \to 2^+$ and $x \to$ infinity, and check whether R takes any finite minimum or maximum on the domain $x > 2$. Give the final range in inequality or interval notation.

(Total for Question 10 is 12 marks)

Mark scheme · IG.M14 Functions: Domain and Range Restrictions

Question 1

- **B1** $f(2) = 5$ cao
- **Answer: $f(2) = 5$**

Question 2

- **B1** $x = 4$ cao
- **Answer: $x = 4$**

Question 3

- **B1** $x \geq -2$ or domain $x \geq -2$ cao
- **Answer: $x \geq -2$**

Question 4

- **M1** identifies $x = -1$ as a value making denominator zero
- **A1** domain = {x : x in R, $x \neq -1$} or written as (-infinity, -1) union (-1, infinity) cao
- **Answer: Excluded $x = -1$; domain {x : x in R, $x \neq -1$}**

Question 5

- **M1** states domain $4 - x \geq 0$, leading to $x \leq 4$
- **A1** gives correct examples, e.g. $x = 0$ inside and $x = 5$ outside, or similar, cao
- **Answer: Domain $x \leq 4$; example inside $x = 0$, outside $x = 5$**

Question 6

- **M1** solves $x^2 - 9 = 0$ to get $x = 3$ and $x = -3$
- **A1** domain = (-infinity, -3) union (-3, 3) union (3, infinity) cao
- **Answer: Excluded $x = -3$ and $x = 3$; domain (-infinity, -3) U (-3, 3) U (3, infinity)**

Question 7

- **M1** states domain as { -1, 0, 1, 2 }
- **A1** states range as {4, 1, 0, -3} cao
- **B1** both domain and range given in correct set notation
- **Answer: Domain = {-1, 0, 1, 2}; Range = {4, 1, 0, -3}**

Question 8

- **M1** evaluates L at the endpoints $x = 0$ and $x = 3$, giving 5 and -1
- **A1** identifies the max and min correctly and orders them for the range, e.g. $-1 \leq L(x) \leq 5$
- **B1** gives the range in correct inequality form, $-1 \leq L(x) \leq 5$ cao
- **Answer: $-1 \leq L(x) \leq 5$**

Question 9

- **M1** completes the square or uses vertex formula to find x-coordinate of vertex $x_v = 2$
- **M1** evaluates Q at the vertex to find $Q(2) = -1$
- **M1** evaluates Q at the endpoints $Q(1) = 0$ and $Q(4) = 3$
- **M1** compares values and identifies minimum and maximum correctly, showing minimum -1 at $x = 2$ and maximum 3 at $x = 4$
- **A1** states the range in correct inequality form, $-1 \leq Q(x) \leq 3$ cao
- **B1** explicit statement that the vertex lies inside the domain and is used to find the minimum, oe
- **A1** correct numerical substitution at endpoints and vertex with no arithmetic error, cao
- **B1** final answer presented clearly as the set of possible outputs, $-1 \leq Q(x) \leq 3$
- **Answer: $-1 \leq Q(x) \leq 3$**

Question 10

- **M1** states x = 2 is excluded from the domain because denominator x - 2 = 0
- **M1** rearranges R(x) into form 2 + 5/(x - 2) or equivalent algebraic manipulation, showing R(x) = 2 + 5/(x - 2)
- **M1** uses form to consider limits: as x -> 2+ then 5/(x - 2) -> +infinity so R -> +infinity, showing no upper bound
- **M1** as x -> +infinity, 5/(x - 2) -> 0 so R -> 2, indicating a horizontal asymptote at y = 2
- **M1** checks monotonic behaviour on x > 2, for example differentiating R or noting 5/(x - 2) is positive and decreases on x > 2, so R decreases from +infinity toward 2
- **A1** concludes that R(x) takes every value greater than 2 on x > 2, so range is (2, infinity) cao
- **B1** correct statement that y = 2 is not attained for finite x > 2, since 5/(x - 2) would have to be 0 which only occurs as x -> infinity
- **A1** final presentation of domain exclusion and range for x > 2, domain x ≠ 2 and range (2, infinity), cao
- **M1** alternative valid reasoning accepted: solves R(x) = y for x and shows for x > 2 this yields y > 2, oe
- **A1** correct algebraic manipulation in alternative route, e.g. solving y = (2x + 1)/(x - 2) leading to x = (2 + 2y)/(y - 2), and concluding y > 2 for x > 2, cao
- **B1** clear statement of excluded value(s) in part (a) and clear linking words in part (b) showing consideration of endpoints and limits
- **A1** final boxed answer: Domain excludes x = 2; Range for x > 2 is (2, infinity) cao
- **Answer: Excluded: x = 2. For domain x > 2, range is (2, infinity).**