HIGHER

2.8 Matrices: Solving Simultaneous Equations With the Inverse Matrix: Test Pack

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 55 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 Write down the 2x2 identity matrix I.

(Total for Question 1 is 1 mark)

2 State one advantage of using the inverse matrix method over substitution when solving two linear equations that come from a word problem, giving a brief reason.

(Total for Question 2 is 1 mark)

3 M = [[x, 3], [12, x]]
Find the values of x for which M has no inverse.

(Total for Question 3 is 4 marks)

4 Given matrix M = [[2, 1], [1, 1]] and vector x = $[p, q]^T$, write the matrix equation M x = c that represents the simultaneous equations 2p + q = 9 and p + q = 5. State c explicitly.

(Total for Question 4 is 1 mark)

© Revision Library | revisionlibrary.co.uk

5 M = [[5, 6], [5, 3]]
Find M^{-1}, the inverse of M.

(Total for Question 5 is 3 marks)

---

6 M = [[-5, -2], [1, -1]]
Find M^{-1}, the inverse of M.

(Total for Question 6 is 3 marks)

---

7 For M = [[3, 2], [1, 1]], calculate det(M). This determinant is used when forming M^{-1}.

(Total for Question 7 is 2 marks)

---

8 Write the simultaneous equations
$-x + 6y = -22$
$6x + 4y = 12$
in the form M [x; y] = c, where M is a 2 x 2 matrix and c is a column vector.

(Total for Question 8 is 2 marks)

© Revision Library | revisionlibrary.co.uk

9 Write the simultaneous equations

$-3y = -9$

$4x - 3y = 15$

in the form M [x; y] = c, where M is a 2 x 2 matrix and c is a column vector.

(Total for Question 9 is 2 marks)

---

10 Write the simultaneous equations

$3y = 6$

$5x + 2y = 24$

in the form M [x; y] = c, where M is a 2 x 2 matrix and c is a column vector.

(Total for Question 10 is 2 marks)

---

11 M = [[x, 3], [2, 4]]

Find the value of x for which M has no inverse.

(Total for Question 11 is 3 marks)

---

12 Write the simultaneous equations

$-3y = -6$

$6x = -6$

in the form M [x; y] = c, where M is a 2 x 2 matrix and c is a column vector.

(Total for Question 12 is 2 marks)

© Revision Library | revisionlibrary.co.uk

13 Write the simultaneous equations

$5x - 2y = -12$

$-x - 2y = -12$

in the form M [x; y] = c, where M is a 2 x 2 matrix and c is a column vector.

(Total for Question 13 is 2 marks)

---

14 Write the simultaneous equations

$-4x + 2y = -6$

$-2x = -8$

in the form M [x; y] = c, where M is a 2 x 2 matrix and c is a column vector.

(Total for Question 14 is 2 marks)

---

15 Write the simultaneous equations

$-x - 2y = 2$

$2x - 3y = -11$

in the form M [x; y] = c, where M is a 2 x 2 matrix and c is a column vector.

(Total for Question 15 is 2 marks)

---

16 For M = [[7, 2], [4, 1]] and c = $[20, 11]^T$, use a calculator to find $x = M^{-1} c$. Give the ordered pair solution and give a one line verification by substitution into the first original equation only.

(Total for Question 16 is 1 mark)

© Revision Library | revisionlibrary.co.uk

17 Given $M = [[4, 1], [3, 2]]$ and $c = [14, 16]^T$, a student claims $x = M^{-1} c$ can be used to solve for x. Write down the first multiplication step that must be done to compute x, showing M^{-1} explicitly in terms of 1/det and the adjugate (do not compute numeric entries).

(Total for Question 17 is 2 marks)

---

18 $M = [[2, 3], [5, 4]]$ and $c = [9, 19]^T$. Use a calculator to find $x = M^{-1} c$ and hence give the solution (x1, x2) for the simultaneous equations $2x1 + 3x2 = 9$ and $5x1 + 4x2 = 19$. Give answers as integers.

(Total for Question 18 is 3 marks)

© Revision Library | revisionlibrary.co.uk

Mark scheme · 2.8 Matrices: Solving Simultaneous Equations With the Inverse Matrix: Test Pack

Question 1

I = [[1, 0], [0, 1]] cao **B1**

Answer: I = [[1, 0], [0, 1]]

Question 2

advantage stated, e.g. inverse method is systematic and faster for multiple similar problems, oe, or gives both unknowns in one calculation, with brief reason **B1**

Answer: The inverse matrix method gives both unknowns in one calculation, which is faster and less error prone for repeated calculations or when coefficients are not simple.

Question 3

M has no inverse when its determinant is 0 **M1**

$x^2 - 36 = 0$ **M1**

$x^2 = 36$, so takes both square roots **M1**

x = 6 or x = -6 **A1**

Answer: x = 6 or x = -6

Question 4

M x = c with $c = [9, 5]^T$ cao **B1**

Answer: M x = [[2,1],[1,1]] $[p, q]^T = [9, 5]^T$

Question 5

determinant = (5)(3) - (6)(5) = -15 **M1**

swaps the leading diagonal and changes the signs of the other two entries **M1**

$M^{-1} = (1/15)[[-3, 6], [5, -5]]$ **A1**

Answer: (1/15)[[-3, 6], [5, -5]]

Question 6

determinant = (-5)(-1) - (-2)(1) = 7 **M1**

swaps the leading diagonal and changes the signs of the other two entries **M1**

$M^{-1} = (1/7)[[-1, 2], [-1, -5]]$ **A1**

Answer: (1/7)[[-1, 2], [-1, -5]]

Question 7

writes det(M) = 3*1 - 2*1 or equivalent **M1**

det(M) = 1 cao **A1**

Answer: det(M) = 1

Question 8

M = [[-1, 6], [6, 4]] **B1**

c = [-22; 12] **B1**

Answer: [[-1, 6], [6, 4]] [x; y] = [-22; 12]

Question 9

M = [[0, -3], [4, -3]] **B1**

c = [-9; 15] **B1**

Answer: [[0, -3], [4, -3]] [x; y] = [-9; 15]

Question 10

M = [[0, 3], [5, 2]] **B1**

c = [6; 24] **B1**

Answer: [[0, 3], [5, 2]] [x; y] = [6; 24]

Question 11

M has no inverse when its determinant is 0 **M1**

4x - 6 = 0 **M1**

x = 3/2 **A1**

Answer: x = 3/2

Question 12

M = [[0, -3], [6, 0]] **B1**

c = [-6; -6] **B1**

Answer: [[0, -3], [6, 0]] [x; y] = [-6; -6]

Question 13

M = [[5, -2], [-1, -2]] **B1**

c = [-12; -12] **B1**

Answer: [[5, -2], [-1, -2]] [x; y] = [-12; -12]

Question 14

M = [[-4, 2], [-2, 0]] **B1**

c = [-6; -8] **B1**

Answer: [[-4, 2], [-2, 0]] [x; y] = [-6; -8]

Question 15

M = [[-1, -2], [2, -3]] **B1**

c = [2; -11] **B1**

Answer: [[-1, -2], [2, -3]] [x; y] = [2; -11]

Question 16

correct pair x = (2,3) and verifies 7*2 + 2*3 = 20; accept correct solution and valid substitution **B1**

Answer: (x1, x2) = (2,3); check: 7*2 + 2*3 = 14 + 6 = 20

Question 17

writes $M^{-1} = 1/\det(M) * [[2, -1], [-3, 4]]$ or equivalent adjugate form **M1**

shows $x = (1/\det(M)) [[2, -1], [-3, 4]] [14,16]^T$ cao **A1**

Answer: $x = (1/\det(M)) [[2, -1], [-3, 4]] [14, 16]^T$

Question 18

uses inverse method: forms M^{-1} and begins multiplication with c, or solves by showing correct matrix multiplication set-up **M1**

carries out multiplication to obtain numerical vector, correct up to a factor of 1/det if applicable **M1**

gives (x1, x2) = (3, 1) cao **A1**

Answer: (x1, x2) = (3, 1)

© Revision Library | revisionlibrary.co.uk