IG.M15 Matrices: Solving Simultaneous Equations With the Inverse Matrix

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 45 minutes

Name: ______________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 Write down the 2x2 identity matrix I.

(Total for Question 1 is 1 mark)

2 Given matrix M = [[2, 1], [1, 1]] and vector x = $[p, q]^T$, write the matrix equation M x = c that represents the simultaneous equations 2p + q = 9 and p + q = 5. State c explicitly.

(Total for Question 2 is 1 mark)

3 For M = [[3, 2], [1, 1]], calculate det(M). This determinant is used when forming M^{-1}.

(Total for Question 3 is 2 marks)

4 Given M = [[4, 1], [3, 2]] and c = $[14, 16]^T$, a student claims x = M^{-1} c can be used to solve for x. Write down the first multiplication step that must be done to compute x, showing M^{-1} explicitly in terms of 1/det and the adjugate (do not compute numeric entries).

(Total for Question 4 is 2 marks)

5 $M = [[2, 3], [5, 4]]$ and $c = [9, 19]^T$. Use a calculator to find $x = M^{-1} c$ and hence give the solution ($x1$, $x2$) for the simultaneous equations $2x1 + 3x2 = 9$ and $5x1 + 4x2 = 19$. Give answers as integers.

(Total for Question 5 is 3 marks)

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6 Write down the condition on det(M) for a 2x2 matrix M to be invertible, and state why this is needed when solving $M x = c$.

(Total for Question 6 is 3 marks)

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7 A small cafe sells two types of sandwich: cheese and ham. In one day the cafe sells 40 sandwiches in total. Each cheese sandwich uses 2 slices of cheese and 1 slice of tomato. Each ham sandwich uses 1 slice of cheese and 2 slices of tomato. That day the kitchen used 62 slices of cheese and 58 slices of tomato. Form two simultaneous equations, write them in matrix form $M x = c$, and use the inverse matrix method to find how many cheese and ham sandwiches were sold. Show your working and check your values by substitution.

(Total for Question 7 is 5 marks)

8 Translate the following word problem into matrix form and solve by the inverse matrix method. A florist arranges bouquets with roses and lilies. Each red bouquet uses 3 roses and 2 lilies. Each white bouquet uses 2 roses and 4 lilies. The florist made 17 bouquets in total and used 44 roses. Form the simultaneous equations, write M x = c, and find how many red and white bouquets were made. Include a check.

(Total for Question 8 is 5 marks)

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9 Solve the simultaneous equations $4a + 5b = 41$ and $3a + 2b = 26$ using matrices. Use the inverse of the coefficient matrix and show your calculation clearly. Give the exact values for a and b and verify by substitution.

(Total for Question 9 is 5 marks)

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10 A shop sells pencils and pens. Two pencils and three pens cost 7 pounds. Four pencils and one pen cost 6 pounds. Let p be the cost of a pencil and q the cost of a pen in pounds. Write the equations in matrix form and use the inverse matrix method to find p and q. Give answers as fractions in simplest form and include a verification.

(Total for Question 10 is 5 marks)

11 Consider the system $x + 2y = 8$ and $6x + 5y = 47$. Use the inverse matrix method to find x and y. Include full calculation of M^{-1} and a final substitution check.

(Total for Question 11 is 5 marks)

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12 Given M = [[1, -1], [2, 3]] and $c = [4, 11]^T$, write down the matrix equation M x = c and state whether det(M) is zero or non zero.

(Total for Question 12 is 1 mark)

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13 For M = [[7, 2], [4, 1]] and $c = [20, 11]^T$, use a calculator to find $x = M^{-1} c$. Give the ordered pair solution and give a one line verification by substitution into the first original equation only.

(Total for Question 13 is 1 mark)

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14 State one advantage of using the inverse matrix method over substitution when solving two linear equations that come from a word problem, giving a brief reason.

(Total for Question 14 is 1 mark)

Mark scheme · IG.M15 Matrices: Solving Simultaneous Equations With the Inverse Matrix

Question 1

- **B1** I = [[1, 0], [0, 1]] cao
- **Answer: I = [[1, 0], [0, 1]]**

Question 2

- **B1** M x = c with c = $[9, 5]^T$ cao
- **Answer: M x = [[2,1],[1,1]] $[p, q]^T = [9, 5]^T$**

Question 3

- **M1** writes det(M) = 3*1 - 2*1 or equivalent
- **A1** det(M) = 1 cao
- **Answer: det(M) = 1**

Question 4

- **M1** writes M^{-1} = 1/det(M) * [[2, -1], [-3, 4]] or equivalent adjugate form
- **A1** shows x = (1/det(M)) [[2, -1], [-3, 4]] $[14,16]^T$ cao
- **Answer: x = (1/det(M)) [[2, -1], [-3, 4]] $[14, 16]^T$**

Question 5

- **M1** uses inverse method: forms M^{-1} and begins multiplication with c, or solves by showing correct matrix multiplication set-up
- **M1** carries out multiplication to obtain numerical vector, correct up to a factor of 1/det if applicable
- **A1** gives (x1, x2) = (3, 1) cao
- **Answer: (x1, x2) = (3, 1)**

Question 6

- **B1** det(M) not equal to 0 stated
- **B1** reason: inverse matrix M^{-1} exists only when det(M) not equal to 0
- **B1** reason: without M^{-1} you cannot form x = M^{-1} c to find a unique solution, oe
- **Answer: det(M) must not equal 0, because only then does M^{-1} exist and x = M^{-1} c gives the unique solution.**

Question 7

- **Level 1** (1-2): Forms at least one correct equation from the worded context and shows partial matrix set-up
- **Level 2** (3-4): Correctly forms both simultaneous equations, writes M x = c, and applies M^-1 to c with some correct arithmetic, producing a candidate solution
- **Level 3** (5): Gives fully correct equations and matrix form, computes M^-1 correctly, finds integer values for both unknowns using x = M^-1 c, and verifies both equations by substitution
- **Indicative content:**
  - Let x = number of cheese sandwiches, y = number of ham sandwiches
  - Equations: x + y = 40 and 2x + y = 62 for cheese slices, or use tomato equation 1x + 2y = 58; any two independent equations are acceptable
  - Matrix form example: [[1,1],[2,1]] [x,y]^T = [40,62]^T or using tomato [40,58]^T; use a consistent pair
  - Compute det = 1*1 - 1*2 = -1 and inverse M^-1 = -1 * [[1,-1],[-2,1]] = [[-1,1],[2,-1]] if using the first matrix, or show correct numeric inverse for the chosen matrix
  - Multiply M^-1 by c to obtain [x,y]^T = [22,18]^T (or equivalent correct pair), and substitute into 2x + y and x + 2y to check totals 62 and 58

Question 8

- **Level 1** (1-2): Forms one correct equation from the context or writes a matrix with one row correct
- **Level 2** (3-4): Forms both simultaneous equations, writes M x = c, and applies inverse with partly correct arithmetic
- **Level 3** (5): Correct matrix equation, computes M^-1 correctly, obtains integer solution and substitutes back to verify both original equations
- **Indicative content:**
  - Let r = red bouquets, w = white bouquets
  - Equations: 3r + 2w = 44 and r + w = 17
  - Matrix form M = [[3,2],[1,1]], x = [r,w]^T, c = [44,17]^T
  - det = 3*1 - 2*1 = 1 so M^-1 = [[1, -2], [-1, 3]]
  - x = M^-1 c = [[1,-2],[-1,3]] [44,17]^T = [44 -34, -44 +51]^T = [10,7]^T, check 3*10 + 2*7 = 44 and 10 + 7 = 17

Question 9

- **Level 1** (1-2): Shows the matrix form or computes det or adjugate partially
- **Level 2** (3-4): Forms M x = c and computes M^-1 with some arithmetic, leading to a candidate solution
- **Level 3** (5): Correct M^-1, correct multiplication giving a and b, and successful substitution into both equations
- **Indicative content:**
  - M = [[4,5],[3,2]], x = [a,b]^T, c = [41,26]^T
  - det = 4*2 - 5*3 = 8 - 15 = -7, adjugate = [[2, -5], [-3, 4]] so M^-1 = -1/7 [[2,-5],[-3,4]]
  - Multiply adjugate by c: [[2,-5],[-3,4]] [41,26]^T = [82 -130, -123 +104] = [-48, -19]
  - Multiply by -1/7 gives [48/7, 19/7], so a = 48/7, b = 19/7, and check by substitution into original equations

Question 10

- **Level 1** (1-2): Forms one equation correctly or writes matrix form with one row incorrect
- **Level 2** (3-4): Forms both equations, computes inverse with partial accuracy, and gets close to the correct fractional answers
- **Level 3** (5): Correct M^-1 used, p and q found as simplified fractions, and substitution verifies both equations
- **Indicative content:**
  - Equations: 2p + 3q = 7 and 4p + q = 6
  - Matrix M = [[2,3],[4,1]], x = [p,q]^T, c = [7,6]^T
  - det = 2*1 - 3*4 = 2 - 12 = -10, adjugate = [[1,-3],[-4,2]]
  - M^-1 = -1/10 [[1,-3],[-4,2]]
  - Multiply and simplify to p = 11/10 (1.10) and q = 8/5 (1.60); check 2*(11/10) + 3*(8/5) = 2.2 + 4.8 = 7 and 4*(11/10) + 8/5 = 4.4 + 1.6 = 6, both correct

Question 11

- **Level 1** (1-2): Forms M x = c and shows some attempt at inverse computation
- **Level 2** (3-4): Computes M^-1 with minor arithmetic slip or obtains one variable correctly
- **Level 3** (5): Correct inverse, correct multiplication giving integer x and y, and both equations verified by substitution
- **Indicative content:**
  - M = [[1,2],[6,5]], x = [x,y]^T, c = [8,47]^T
  - det = 1*5 - 2*6 = 5 - 12 = -7, adjugate = [[5, -2], [-6, 1]]
  - M^-1 = -1/7 [[5,-2],[-6,1]]
  - Multiply adjugate by c: [[5,-2],[-6,1]] [8,47]^T = [40 -94, -48 +47] = [-54, -1], multiply by -1/7 gives [54/7, 1/7], so x = 54/7, y = 1/7, and check in both original equations

Question 12

- **B1** M x = c written correctly and det(M) = 1*3 - (-1)*2 = 3 +2 = 5 stated non zero
- **Answer: M x = [[1,-1],[2,3]] $[x]^T$ = $[4,11]^T$ and det(M) = 5, non zero**

Question 13

- **B1** correct pair x = (2,3) and verifies 7*2 + 2*3 = 20; accept correct solution and valid substitution
- **Answer: (x1, x2) = (2,3); check: 7*2 + 2*3 = 14 + 6 = 20**

Question 14

- **B1** advantage stated, e.g. inverse method is systematic and faster for multiple similar problems, oe, or gives both unknowns in one calculation, with brief reason
- **Answer: The inverse matrix method gives both unknowns in one calculation, which is faster and less error prone for repeated calculations or when coefficients are not simple.**