HIGHER

3.5 Circle Theorems: Angle Properties: Test Pack

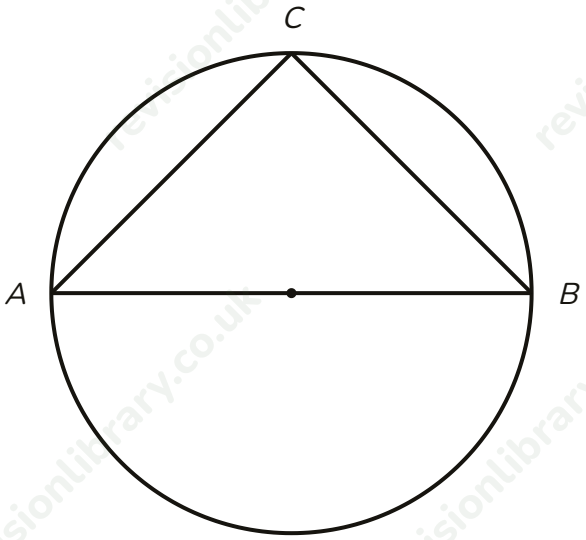
Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A semicircle has diameter AB. In the semicircle the angle ACB is marked at point C on the circumference. Find angle ACB and name the theorem used.



(Total for Question 1 is 1 mark)

2 In a circle with centre O, the angle at the centre AOB is shown as 120 degrees. State the angle at the circumference subtended by the same arc AB in the same circle.

(Total for Question 2 is 1 mark)

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3 A circle passes through four points A, B, C and D, which lie on the circumference in that order going round the circle, so that ABCD is a cyclic quadrilateral. Angle DAB = 75 degrees and angle ABC = 90 degrees. The diagonal AC is drawn, splitting angle BCD into angle BCA (the part next to B) and angle ACD (the part next to D), with angle ACD = 40 degrees. The tangent to the circle at D meets the straight line through A and B, extended beyond A, at the point E, so that E, A and B lie in that order on the line.

(a) Find angle BCD and angle CDA, the other two angles of the cyclic quadrilateral ABCD, stating the circle theorem you use. **(4)**

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(b) Find angle BCA. Then find angle ABD, the angle between BA and the diagonal BD, stating the circle theorem you use. **(6)**

.....

(c) The tangent at D meets line AB extended beyond A at E, as described above. Find angle EDA, the angle between the tangent ED and the chord DA, stating the circle theorem you use. Hence find angle AED. **(8)**

.....

(Total for Question 3 is 18 marks)

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4 A circle has centre O. From an external point P, two tangents touch the circle at A and B, so that PA and PB are tangents to the circle. The angle AOB, between the two radii OA and OB, is 100 degrees.

(a) State the size of angle OAP and angle OBP, giving a reason. Hence explain why PA = PB. **(4)**

(b) Find angle APB, the angle between the two tangents at P. A point Q lies on the major arc AB (the arc on the far side of the circle from P). Find angle AQB, stating the circle theorem you use. **(6)**

(c) Show that angle PAB = 50 degrees using two different methods: (i) using the fact that triangle PAB is isosceles, and (ii) using the alternate segment theorem together with your answer to part (b). Explain why the two methods agree. **(8)**

(Total for Question 4 is 18 marks)

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5 In a circle with centre O, chord AB subtends angle AOB = 80 degrees at the centre. On the circumference point C lies on the same arc AB. Calculate angle ACB and give the theorem used.

(Total for Question 5 is 2 marks)

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Mark scheme · 3.5 Circle Theorems: Angle Properties: Test Pack

Question 1

angle ACB = 90 degrees and theorem named, cao **B1**

Answer: 90 degrees

Question 2

correct angle 60 degrees, cao **B1**

Answer: 60 degrees

Question 3

(a) angle BCD = 180 - angle DAB, or equivalent method **M1**

(a) angle BCD = 105 degrees cao, with theorem named: opposite angles in a cyclic quadrilateral sum to 180 degrees **A1**

(a) angle CDA = 180 - angle ABC, or equivalent method **M1**

(a) angle CDA = 90 degrees cao, theorem named as above **A1**

(a) Answer: angle BCD = 105 degrees, angle CDA = 90 degrees

(b) angle BCA = angle BCD - angle ACD **M1**

(b) angle BCA = 65 degrees cao **A1**

(b) identifies that chord AD is subtended by angle ACD at C and angle ABD at B, with B and C on the same arc relative to AD **M1**

(b) states angle ABD = angle ACD **A1**

(b) angle ABD = 40 degrees cao **B1**

(b) names the theorem: angles in the same segment of a circle are equal **B1**

(b) Answer: angle BCA = 65 degrees, angle ABD = 40 degrees

(c) identifies angle EDA as the tangent-chord angle for chord DA, equal to the angle in the alternate segment (angle ABD or angle ACD from part (b)) **M1**

(c) angle EDA = 40 degrees, theorem named: alternate segment theorem **A1**

(c) angle DAE = 180 - angle DAB, since E, A, B are collinear **M1**

(c) angle DAE = 105 degrees cao **A1**

(c) uses angle sum of triangle ADE: angle AED = 180 - angle DAE - angle EDA **M1**

(c) correct substitution 180 - 105 - 40 **A1**

(c) angle AED = 35 degrees cao **A1**

(c) names the angle sum of a triangle (180 degrees) used for the final step **B1**

(c) Answer: angle EDA = 40 degrees, angle AED = 35 degrees

Question 4

(a) angle OAP = angle OBP = 90 degrees **B1**

(a) names theorem: the radius to the point of contact is perpendicular to the tangent **B1**

(a) states PA = PB **B1**

(a) names theorem: tangents drawn from the same external point to a circle are equal in length **B1**

(a) Answer: angle OAP = angle OBP = 90 degrees; PA = PB

(b) forms angle AOB + angle OAP + angle APB + angle OBP = 360, using angles in a quadrilateral **M1**

(b) names the reasoning: angles in a quadrilateral sum to 360 degrees **B1**

(b) angle APB = 360 - 100 - 90 - 90 = 80 degrees cao **A1**

(b) uses angle AQB = angle AOB / 2, since Q is on the major arc **M1**

(b) names theorem: the angle at the centre is twice the angle at the circumference on the same arc **B1**

(b) angle AQB = 50 degrees cao **A1**

(b) Answer: angle APB = 80 degrees, angle AQB = 50 degrees

(c) uses angle sum of triangle PAB: angle PAB + angle PBA + angle APB = 180 **M1**

(c) uses PA = PB, so triangle PAB is isosceles and angle PAB = angle PBA **M1**

(c) method (i): angle PAB = (180 - 80) / 2 = 50 degrees **A1**

(c) identifies tangent PA and chord AB, with the angle between them equal to the angle in the alternate segment **M1**

(c) states this alternate segment angle equals angle AQB, found in part (b) **A1**

(c) method (ii): angle PAB = 50 degrees **A1**

(c) names theorem: alternate segment theorem **B1**

(c) explains the two methods agree because each correctly identifies the same angle, one via isosceles triangle base angles and one via the alternate segment theorem **B1**

(c) Answer: angle PAB = 50 degrees, confirmed by two independent methods

Question 5

uses relationship angle at circumference = 1/2 angle at centre or divides 80 by 2 **M1**

angle ACB = 40 degrees cao, with theorem named **A1**

Answer: 40 degrees

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