HIGHER

IG.M16 Circle Theorems: Angle Properties

EDEXCEL 4MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 In a circle with centre O, the angle at the centre AOB is shown as 120 degrees. State the angle at the circumference subtended by the same arc AB in the same circle and name the theorem used, context: angle at centre.

(Total for Question 1 is 1 mark)

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2 A semicircle has diameter AB. In the semicircle the angle ACB is marked at point C on the circumference. Find angle ACB and name the theorem used, context: angle in a semicircle.

(Total for Question 2 is 1 mark)

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3 In a circle, points A, B, C, D lie on the circumference with arc AB fixed. Angles ACB and ADB are given as equal. State the theorem that explains why these two angles are equal, context: angles in the same segment.

(Total for Question 3 is 1 mark)

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4 In a circle with centre O, chord AB subtends angle AOB = 80 degrees at the centre. On the circumference point C lies on the same arc AB. Calculate angle ACB and give the theorem used, context: angle at centre to circumference calculation.

(Total for Question 4 is 2 marks)

5 A circle has diameter PQ = 14 cm. Point R lies on the circle with PR and RQ forming the triangle PRQ in the semicircle. Calculate PR times RQ if PR = 6 cm and RQ = 8 cm and state the angle in a semicircle result for angle PRQ, context: semicircle diameter and triangle side lengths.

(Total for Question 5 is 2 marks)

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6 In circle with centre O, tangent at T meets the circle at T and radius OT is drawn. Give a reason why OT is perpendicular to the tangent at T and name the theorem used, context: tangent perpendicularity to radius.

(Total for Question 6 is 2 marks)

7 A circle with centre O has a cyclic quadrilateral ABCD drawn on its circumference. The angles at A and C are labelled 70 degrees and x degrees respectively. A tangent at D is drawn and meets the extension of AB at point E outside the circle. Using the diagram: find x, then find angle AED at E, show the theorems used for each step, and give reasons. Context: cyclic quadrilateral and alternate segment theorem combined for an extended proof calculation.

Figure (to be drawn): *A circle with points A, B, C, D on the circumference forming a cyclic quadrilateral ABCD. Angle at A is 70 degrees. Angle at C is marked x. A tangent at D is drawn and meets the extension of AB beyond B at E. Lines: AB extended to E, tangent at D meeting AB produced at E. Centre O is shown but not needed numerically.*

(Total for Question 7 is 18 marks)

8 A circle has centre O. From an external point P two tangents PA and PB touch the circle at A and B respectively. A line through P meets the circle at points Q and R so that Q is nearer to P. The angle QPA is given as 50 degrees and angle AOB, the central angle between radii OA and OB, is 100 degrees. Using the diagram: show that PA = PB, explain why angle AOB is twice angle AQB, calculate angle AQB and then find angle PRB, giving the theorems used and reasons in each step. Context: equal tangents, angle at centre twice circumference and angles in same segment together.

Figure (to be drawn): A circle with centre O. External point P has two tangents PA and PB touching the circle at A and B. A line through P meets the circle at Q and R with Q nearer to P. Angle QPA is 50 degrees. The central angle AOB is 100 degrees. Radii OA and OB are drawn to the points of contact A and B.

(Total for Question 8 is 18 marks)

Mark scheme · IG.M16 Circle Theorems: Angle Properties

Question 1

- **B1** correct angle 60 degrees, cao
- **Answer: 60 degrees**

Question 2

- **B1** angle ACB = 90 degrees and theorem named, cao
- **Answer: 90 degrees**

Question 3

- **B1** angles in the same segment theorem named, cao
- **Answer: Angles in the same segment are equal**

Question 4

- **M1** uses relationship angle at circumference = 1/2 angle at centre or divides 80 by 2
- **A1** angle ACB = 40 degrees cao, with theorem named
- **Answer: 40 degrees**

Question 5

- **M1** multiplies 6 and 8 to get 48 or states right angle using semicircle theorem
- **A1** answers: product 48 and angle PRQ = 90 degrees, both correct and theorem named
- **Answer: PR times RQ = 48, angle PRQ = 90 degrees**

Question 6

- **B1** states OT is perpendicular to tangent because radius to point of contact is perpendicular to tangent
- **B1** names theorem: tangent is perpendicular to radius at point of contact
- **Answer: OT is perpendicular to the tangent at T because the radius to the point of contact is perpendicular to the tangent, theorem named**

Question 7

- **Level 1** (1-4): Partial identification of relevant theorems and one correct simple angle relation, with limited correctness of reasoning
- **Level 2** (5-8): Uses cyclic quadrilateral property to relate opposite angles and finds x or angle AED with some correct reasons, but may omit the alternate segment theorem or the naming of a theorem
- **Level 3** (9-12): Correctly finds x using cyclic quadrilateral property and gives a reason, and makes a correct attempt to use the tangent relation or alternate segment theorem to find angle AED with mostly correct working
- **Level 4** (13-18): Fully correct chain of reasoning: uses cyclic quadrilateral theorem to find x accurately, and applies the alternate segment theorem and tangent properties to find angle AED, naming the theorems and giving clear reasons linking each step
- **Indicative content:**
  - Use cyclic quadrilateral property: opposite angles sum to 180 degrees, so x + 70 = 180, giving x = 110 degrees
  - State the theorem name: opposite angles in a cyclic quadrilateral sum to 180 degrees
  - Note relation between tangent and alternate segment: angle between tangent and chord equals angle in the opposite arc
  - Identify the chord corresponding to angle AED: chord AD or chord AB depending on diagram orientation and show how the angle between tangent at D and chord DA equals angle in the opposite segment at B or C
  - Deduce angle AED from the alternate segment theorem, giving the numeric value consistent with x and the given angles, and name the theorem
  - Explain each step briefly and clearly so the sequence x found then angle at E follows logically

Question 8

- **Level 1** (1-4): Identifies simple facts such as two tangents from an external point are equal or that angle at centre relates to angle at circumference, but with limited linking or inaccurate numerics
- **Level 2** (5-8): Explains PA = PB with a correct brief reason and correctly states that angle at centre is twice angle at circumference, with partial calculation of AQB or PRB
- **Level 3** (9-12): Shows PA = PB with a clear reason, uses angle at centre theorem to find AQB = 50 degrees and completes calculation for PRB with minor omissions in justification
- **Level 4** (13-18): Fully correct solution: proves PA = PB with reason, uses AOB = 100 degrees to deduce AQB = 50 degrees naming the angle at centre theorem, and finds PRB using angles in the same segment or tangent-chord relations, with clear reasons and correct numerics
- **Indicative content:**
  - PA = PB because tangents from an external point to a circle have equal lengths, name the theorem
  - PA and OA make a right angle at A since radius to point of contact is perpendicular to tangent, similarly for PB and OB, implying triangles OAP and OBP are congruent in the RHS sense giving PA = PB or give reason using equal tangents theorem
  - Angle at centre AOB = 100 degrees implies angle AQB, the angle at the circumference subtended by arc AB, is 50 degrees using angle at centre is twice angle at circumference
  - State AQB = 50 degrees numerically and name the theorem
  - Find angle PRB by noting Q and R are points on the same chord or arc: use angles in the same segment or consider triangle relations with known angles, and compute PRB numerically consistent with given 50 degrees
  - Name each theorem used: equal tangents, tangent perpendicular to radius if used, angle at centre twice angle at circumference, angles in the same segment