HIGHER

5.2 Calculus in Kinematics: Displacement, Velocity and Acceleration: 2-hour Pack

EDEXCEL 4MA1 · Calculator allowed · about 120 minutes

Total Marks

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A particle moves in a straight line. Its displacement from the fixed point O after t seconds is $s = 2t^3 - 12t^2 + 18t$ metres, for $0 \leq t \leq 4$.
Find the total distance the particle travels in the first 4 seconds.

(Total for Question 1 is 6 marks)

2 A particle moves in a straight line. Its displacement s metres from a fixed point O after t seconds is $s = t^3 + 2t^2 - 2t$.
Find the velocity of the particle when $t = 1$.

(Total for Question 2 is 2 marks)

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3 A particle moves in a straight line. Its displacement from a fixed point O after t seconds is $s = t^3 - 3t^2 + 3$ metres.
Find the time at which the acceleration of the particle is zero, and find the velocity of the particle at this time.

(Total for Question 3 is 5 marks)

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4 A particle P moves along a straight line. Its displacement s metres from a fixed point O after t seconds is $s = t^3 - 12t^2 + 36t + 4$, for $t \geq 0$.
Find the times at which P is instantaneously at rest, and the displacement of P from O at each of these times.

(Total for Question 4 is 5 marks)

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5 A particle moves in a straight line. At time t seconds its velocity is $v = 9t^2 + 12t + 8$ m/s. When $t = 0$ the particle is -3 m from a fixed point O.
Find the displacement of the particle from O when $t = 3$.

(Total for Question 5 is 4 marks)

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6 A particle moves in a straight line. Its displacement from the fixed point O after t seconds is $s = -t^3 + 9t^2 - 15t$ metres, for $0 \leq t \leq 6$.
Find the total distance the particle travels in the first 6 seconds.

(Total for Question 6 is 6 marks)

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7 A particle moves in a straight line. Its displacement s metres from a fixed point O after t seconds is $s = -2t^2 - 3t$.
Find the velocity of the particle when $t = 1$.

(Total for Question 7 is 2 marks)

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8 A car accelerates in a straight line. At time t seconds its acceleration is $a = 4t - 1$ m/s^2. When $t = 0$ the speed of the car is 4 m/s.
Find the velocity of the car when $t = 3$.

(Total for Question 8 is 4 marks)

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9 A particle moves in a straight line. Its displacement from a fixed point O after t seconds is $s = t^3 - 6t^2 + 9t + 4$ metres.
Find the time at which the acceleration of the particle is zero, and find the velocity of the particle at this time.

(Total for Question 9 is 5 marks)

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10 A model rocket moves vertically so that its displacement above the launch pad in metres is $s = t^3 - 4t^2 + t$ for time t seconds after launch.

(a) Find the velocity function $v = ds/dt$ for the rocket. **(2)**

(b) Find the acceleration at t = 2 s and state whether the rocket is speeding up or slowing down at t = 2 s, with a brief reason. **(4)**

(Total for Question 10 is 6 marks)

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11 A particle moves in a straight line. Its displacement from a fixed point O after t seconds is $s = t^3 - 15t^2 + 72t$ metres.
Find the time at which the acceleration of the particle is zero, and find the velocity of the particle at this time.

(Total for Question 11 is 5 marks)

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12 A car accelerates in a straight line. At time t seconds its acceleration is $a = 3t - 7$ m/s^2. When $t = 0$ the speed of the car is -3 m/s.
Find the velocity of the car when $t = 4$.

(Total for Question 12 is 4 marks)

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13 A particle moves in a straight line. At time t seconds its velocity is $v = 8t - 4$ m/s. When $t = 0$ the particle is 6 m from a fixed point O.
Find the displacement of the particle from O when $t = 2$.

(Total for Question 13 is 4 marks)

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14 A particle moves in a straight line. Its displacement s metres from a fixed point O after t seconds is $s = -2t^3 - t^2 - t$.
Find the velocity of the particle when $t = 4$.

(Total for Question 14 is 2 marks)

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15 A car accelerates in a straight line. At time t seconds its acceleration is $a = 3t + 2$ m/s^2. When $t = 0$ the speed of the car is 7 m/s.
Find the velocity of the car when $t = 2$.

(Total for Question 15 is 4 marks)

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16 A particle has displacement $s = 0.5t^3 - 3t^2 + 2t$ metres after t seconds.

(a) Find expressions for velocity v and acceleration a. **(2)**

.....

(b) Find the times $t \geq 0$ when the particle is instantaneously at rest ($v = 0$). **(2)**

.....

(c) For each time found, state the sign of a and whether the particle is momentarily speeding up or slowing down, with a short reason. **(2)**

(Total for Question 16 is 6 marks)

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17 An experimental glider has displacement $s = 4t^2 - t^3 + 6t$ metres from a reference point after t seconds.

(a) Find expressions for v and a. **(2)**

(b) Find the time(s) $t \geq 0$ when the glider is instantaneously at rest ($v = 0$). **(3)**

(c) For each time found, give a value of a and a one line interpretation of the motion immediately after that instant. **(2)**

(Total for Question 17 is 7 marks)

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18 A particle P moves along a straight line. Its displacement s metres from a fixed point O after t seconds is $s = -t^3 + 9t^2 - 15t + 7$, for $t \geq 0$.
Find the times at which P is instantaneously at rest, and the displacement of P from O at each of these times.

(Total for Question 18 is 5 marks)

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Mark scheme · 5.2 Calculus in Kinematics: Displacement, Velocity and Acceleration: 2-hour Pack

A correct answer earns full marks unless the question asks for working, a method, proof or explanation.

Question 1

$v = ds/dt = 6t^2 - 24t + 18$	**M1**
sets v = 0 and solves to find when the particle changes direction	**M1**
t = 1 and t = 3	**A1**
finds s at t = 0, 1, 3 and 4: 0, 8, 0, 8	**M1**
adds the magnitudes of the three separate displacements	**M1**
total distance = 24 m cao	**A1**

Answer: Total distance = 24 m

Question 2

$v = ds/dt = 3t^2 + 4t - 2$	**M1**
v = 5 m/s cao	**A1**

Answer: v = 5 m/s

Question 3

$v = ds/dt = 3t^2 - 6t$	**M1**
$a = dv/dt = 6t - 6$	**M1**
sets a = 0 and solves	**M1**
t = 1 seconds	**A1**
v = -3 m/s cao	**A1**

Answer: t = 1 s, v = -3 m/s

Question 4

$v = ds/dt = 3t^2 - 24t + 36$	**M1**
sets v = 0 and solves the quadratic	**M1**
t = 2 and t = 6	**A1**
substitutes both times into the expression for s	**M1**
s = 36 m when t = 2 and s = 4 m when t = 6	**A1**

Answer: t = 2, s = 36 m; t = 6, s = 4 m

Question 5

integrates the velocity expression with respect to t	**M1**
$s = 3t^3 + 6t^2 + 8t + c$ with all terms correct	**A1**
uses s = -3 when t = 0 to find c = -3	**M1**
s = 156 m cao	**A1**

Answer: s = 156 m

Question 6

$v = ds/dt = -3t^2 + 18t - 15$	**M1**
sets v = 0 and solves to find when the particle changes direction	**M1**
t = 1 and t = 5	**A1**
finds s at t = 0, 1, 5 and 6: 0, -7, 25, 18	**M1**
adds the magnitudes of the three separate displacements	**M1**
total distance = 46 m cao	**A1**

Answer: Total distance = 46 m

Question 7

$v = ds/dt = -4t - 3$	**M1**
v = -7 m/s cao	**A1**

Answer: v = -7 m/s

Question 8

integrates the acceleration with respect to t	**M1**
$v = 2t^2 - t + c$	**A1**
uses v = 4 when t = 0 to find c = 4	**M1**
v = 19 m/s cao	**A1**

Answer: v = 19 m/s

Question 9

$v = ds/dt = 3t^2 - 12t + 9$	**M1**
$a = dv/dt = 6t - 12$	**M1**
sets a = 0 and solves	**M1**
t = 2 seconds	**A1**
v = -3 m/s cao	**A1**

Answer: t = 2 s, v = -3 m/s

Question 10

(a) differentiates terms of s to form v, eg $3t^2 - 8t + 1$ seen	**M1**
(a) $v = 3t^2 - 8t + 1$ cao	**A1**

(a) Answer: $v = 3t^2 - 8t + 1$

(b) differentiates v to get $a = 6t - 8$	**M1**
(b) substitutes t = 2 to find a(2) = 4	**M1**
(b) $a = 4$ m/s^2 cao	**A1**
(b) correct statement about speeding up or slowing down with reason, eg v(2) = 3(4) - 16 + 1 = -3 m/s so velocity is negative while acceleration is positive, therefore the rocket is slowing down at t = 2 s, cao	**A1**

(b) Answer: a = 4 m/s^2; the rocket is slowing down at t = 2 s, because v(2) = -3 m/s and a is positive so velocity and acceleration have opposite signs

Question 11

$v = ds/dt = 3t^2 - 30t + 72$	**M1**
$a = dv/dt = 6t - 30$	**M1**
sets a = 0 and solves	**M1**
t = 5 seconds	**A1**
v = -3 m/s cao	**A1**

Answer: t = 5 s, v = -3 m/s

Question 12

integrates the acceleration with respect to t	**M1**
$v = (3/2)t^2 - 7t + c$	**A1**
uses v = -3 when t = 0 to find c = -3	**M1**
v = -7 m/s cao	**A1**

Answer: v = -7 m/s

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Question 13

integrates the velocity expression with respect to t **M1**

$s = 4t^2 - 4t + c$ with all terms correct **A1**

uses $s = 6$ when $t = 0$ to find $c = 6$ **M1**

$s = 14$ m cao **A1**

Answer: s = 14 m

Question 14

$v = ds/dt = -6t^2 - 2t - 1$ **M1**

$v = -105$ m/s cao **A1**

Answer: v = -105 m/s

Question 15

integrates the acceleration with respect to t **M1**

$v = (3/2)t^2 + 2t + c$ **A1**

uses $v = 7$ when $t = 0$ to find $c = 7$ **M1**

$v = 17$ m/s cao **A1**

Answer: v = 17 m/s

Question 16

(a) differentiates s to get $v = 1.5t^2 - 6t + 2$ or equivalent **M1**

(a) $a = 3t - 6$ cao **A1**

(a) Answer: $v = 1.5t^2 - 6t + 2$; $a = 3t - 6$

(b) sets $1.5t^2 - 6t + 2 = 0$ and attempts to solve, e.g. multiply by 2 to $3t^2 - 12t + 4 = 0$ **M1**

(b) $t = (12 \pm \sqrt{144 - 48})/6 = (12 \pm \sqrt{96})/6 = (12 \pm 4*\sqrt{6})/6$, giving $t = 2 \pm (2/3)\sqrt{6}$, both values evaluated and non-negative, cao **A1**

(b) Answer: $t = 2 \pm (2/3)\sqrt{6}$ seconds (both are ≥ 0)

(c) evaluates a at each time correctly, recognising sign, eg $a(0.367) = 3(0.367) - 6 < 0$ and $a(3.633) = 3(3.633) - 6 > 0$ **M1**

(c) states correctly: at the smaller root $a < 0$ so since $v = 0$ and $a < 0$ the particle will start moving in the negative direction, speed will increase in the negative direction (interpreted as speeding up if velocity and acceleration same sign, here acceleration negative and velocity changes to negative so speed increases), and at the larger root $a > 0$ so particle will start moving in the positive direction and speed will increase in the positive direction, cao **A1**

(c) Answer: At t approx 0.367 s, a < 0 so the particle will start accelerating in the negative direction and speed will increase in that direction. At t approx 3.633 s, a > 0 so it will accelerate in the positive direction and speed will increase in the positive direction.

Question 17

(a) differentiates s to get $v = 8t - 3t^2 + 6$ or equivalent and $a = 8 - 6t$ **M1**

(a) $v = 8t - 3t^2 + 6$ and $a = 8 - 6t$ cao **A1**

(a) Answer: $v = 8t - 3t^2 + 6$; $a = 8 - 6t$

(b) sets $8t - 3t^2 + 6 = 0$ and attempts rearrangement, e.g. $3t^2 - 8t - 6 = 0$ **M1**

(b) uses quadratic formula or factorises correctly to find roots **M1**

(b) $t = (8 \pm \sqrt{64 + 72})/(6) = (8 \pm \sqrt{136})/6 = (8 \pm 2*\sqrt{34})/6$, giving two real roots, numeric approximations t approx -0.610 (reject) and t approx 3.277 s, so t = 3.277 s (to 3 dp) accepted cao **A1**

(b) Answer: t approx 3.277 s (the only non-negative root)

(c) evaluates a at the non-negative root correctly, e.g. $a(3.277) = 8 - 6(3.277)$ approx -11.66 **M1**

(c) gives correct interpretation: a is negative so after coming to rest the glider will accelerate in the negative direction and speed will increase in that direction, cao **A1**

(c) Answer: a(3.277) approx -11.66 m/s^2; since a < 0 the glider will begin to move in the negative direction and its speed in that direction will increase immediately after t = 3.277 s.

Question 18

$v = ds/dt = -3t^2 + 18t - 15$ **M1**

sets $v = 0$ and solves the quadratic **M1**

$t = 1$ and $t = 5$ **A1**

substitutes both times into the expression for s **M1**

$s = 0$ m when $t = 1$ and $s = 32$ m when $t = 5$ **A1**

Answer: t = 1, s = 0 m; t = 5, s = 32 m

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