HIGHER

5.2 Calculus in Kinematics: Displacement, Velocity and Acceleration: Test Pack

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 60 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 A toy car moves along a straight track. Its displacement s metres from the start at time t seconds is given by $s = 4t^2$. Find the velocity $v = ds/dt$ of the toy car.

(Total for Question 1 is 2 marks)

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2 A car accelerates in a straight line. At time t seconds its acceleration is $a = 6t - 5$ m/s^2. When $t = 0$ the speed of the car is -4 m/s.
Find the velocity of the car when $t = 5$.

(Total for Question 2 is 4 marks)

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3 A particle moves in a straight line. Its displacement from the fixed point O after t seconds is $s = t^3 - 15t^2 + 72t$ metres, for $0 \leq t \leq 7$.
Find the total distance the particle travels in the first 7 seconds.

(Total for Question 3 is 6 marks)

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4 A particle moves in a straight line. Its displacement s metres from a fixed point O after t seconds is $s = 4t^3 - 3t^2 - 3t$.
Find the velocity of the particle when $t = 1$.

(Total for Question 4 is 2 marks)

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5 A particle moves so that its displacement from a fixed point is $s = 20t - 4t^2$ metres after t seconds.

(a) Find the velocity function v and hence find the time(s) when the particle is instantaneously at rest ($v = 0$). **(3)**

(b) Find the acceleration a and state whether the particle is speeding up or slowing down at $t = 3$ s, with a brief reason. **(4)**

(Total for Question 5 is 7 marks)

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6 A car accelerates in a straight line. At time t seconds its acceleration is $a = 3t - 1$ m/s^2. When $t = 0$ the speed of the car is -2 m/s.
Find the velocity of the car when $t = 4$.

(Total for Question 6 is 4 marks)

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7 A particle P moves along a straight line. Its displacement s metres from a fixed point O after t seconds is $s = t^3 - 9t^2 + 24t + 6$, for $t \geq 0$.

Find the times at which P is instantaneously at rest, and the displacement of P from O at each of these times.

(Total for Question 7 is 5 marks)

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8 A motorbike's displacement along a straight road is $s = -2t^3 + 15t^2 - 24t$ metres after t seconds.

(a) Find the acceleration function a(t). **(2)**

.....

(b) Find times $t \geq 0$ when acceleration is zero. **(2)**

.....

(c) State the sign of acceleration for $t < 2.5$ and give a brief implication for the motorbike's motion in that interval. **(3)**

(Total for Question 8 is 7 marks)

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9 A commuter walking along a straight platform has displacement $s = 50 - 5t - t^2$ metres from a fixed bench after t seconds. Find the time when the commuter is instantaneously at rest and state the direction of motion immediately after that time. Give a short reason.

(Total for Question 9 is 4 marks)

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Mark scheme · 5.2 Calculus in Kinematics: Displacement, Velocity and Acceleration: Test Pack

A correct answer earns full marks unless the question asks for working, a method, proof or explanation.

Question 1

differentiates s with respect to t, eg writes $d/dt(4t^2)$ or 8t seen	**M1**
v = 8t cao	**A1**

Answer: v = 8t

Question 2

integrates the acceleration with respect to t	**M1**
$v = 3t^2 - 5t + c$	**A1**
uses v = -4 when t = 0 to find c = -4	**M1**
v = 46 m/s cao	**A1**

Answer: v = 46 m/s

Question 3

$v = ds/dt = 3t^2 - 30t + 72$	**M1**
sets v = 0 and solves to find when the particle changes direction	**M1**
t = 4 and t = 6	**A1**
finds s at t = 0, 4, 6 and 7: 0, 112, 108, 112	**M1**
adds the magnitudes of the three separate displacements	**M1**
total distance = 120 m cao	**A1**

Answer: Total distance = 120 m

Question 4

$v = ds/dt = 12t^2 - 6t - 3$	**M1**
v = 3 m/s cao	**A1**

Answer: v = 3 m/s

Question 5

(a) differentiates s to find v = 20 - 8t or equivalent	**M1**
(a) sets 20 - 8t = 0 and rearranges to 8t = 20	**M1**
(a) t = 2.5 seconds cao	**A1**

(a) Answer: v = 20 - 8t; particle at rest at t = 2.5 s

(b) differentiates v to find $a = dv/dt = -8$	**M1**
(b) substitutes t = 3 into v and/or uses sign of v at t = 3 or evaluates a at t = 3 (a is constant) to compare signs	**M1**
(b) $a = -8$ m/s^2 cao	**A1**
(b) particle is speeding up at t = 3 s, with correct reason linking signs, eg velocity at t = 3 is 20 - 24 = -4 m/s and acceleration is -8 m/s^2, both negative (the same sign), so the magnitude of the velocity is increasing	**A1**

(b) Answer: a = -8 m/s^2; at t = 3 s the particle is speeding up (v = -4 m/s and a = -8 m/s^2 have the same sign)

Question 6

integrates the acceleration with respect to t	**M1**
$v = (3/2)t^2 - t + c$	**A1**
uses v = -2 when t = 0 to find c = -2	**M1**
v = 18 m/s cao	**A1**

Answer: v = 18 m/s

Question 7

$v = ds/dt = 3t^2 - 18t + 24$	**M1**
sets v = 0 and solves the quadratic	**M1**
t = 2 and t = 4	**A1**
substitutes both times into the expression for s	**M1**
s = 26 m when t = 2 and s = 22 m when t = 4	**A1**

Answer: t = 2, s = 26 m; t = 4, s = 22 m

Question 8

(a) differentiates twice, eg $v = -6t^2 + 30t - 24$ and a = -12t + 30 shown	**M1**
(a) a = -12t + 30 cao	**A1**

(a) Answer: a = -12t + 30

(b) sets -12t + 30 = 0 and solves to give t = 30/12	**M1**
(b) t = 2.5 seconds cao	**A1**

(b) Answer: t = 2.5 s

(c) evaluates a for t < 2.5, e.g. picks t = 0 or states -12t + 30 > 0 when t < 2.5	**M1**
(c) states sign correctly, eg a > 0 for t < 2.5	**M1**
(c) gives correct implication, eg the velocity of the motorbike is increasing while a > 0 (it is speeding up only while v > 0, that is for 1 < t < 2.5, and slowing down while v < 0, that is for t < 1), with a brief correct reason linking sign of a to change in velocity	**A1**

(c) Answer: For t < 2.5 s, a > 0, so the velocity of the motorbike is increasing in that interval (it is slowing down for t < 1, where v < 0, and speeding up for 1 < t < 2.5, where v > 0).

Question 9

differentiates s to get v = -5 - 2t or equivalent	**M1**
sets -5 - 2t = 0 and solves for t, e.g. 2t = -5	**M1**
t = -2.5 s cao, or rejects as not physically valid if only t ≥ 0 allowed and gives correct alternative statement	**A1**
direction immediately after that time: for physical t ≥ 0 there is no positive time when at rest, or if candidate accepts negative time then for t just greater than -2.5 v is negative so motion is in the negative direction, with a concise correct reason	**A1**

Answer: v = -5 - 2t so v = 0 at t = -2.5 s (not in the physical domain t ≥ 0); therefore the commuter is never instantaneously at rest for t ≥ 0. If negative time is considered, immediately after t = -2.5 s the velocity is negative, so motion is in the negative direction.

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