HIGHER

IG.M19 Calculus in Kinematics: Displacement, Velocity and Acceleration

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 60 minutes

Name: ______________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 A toy car moves along a straight track. Its displacement s metres from the start at time t seconds is given by $s = 4t^2$. Find the velocity $v = ds/dt$ of the toy car.

(Total for Question 1 is 2 marks)

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2 A cyclist's displacement in metres along a straight road is modelled by $s = 3t^3 - 5t$ for time t seconds. (a) Find the velocity $v = ds/dt$. (b) Calculate the velocity when $t = 2$ seconds.

(a) Find the velocity function $v = ds/dt$ for the cyclist. **(2)**

....

(b) Calculate the velocity when $t = 2$ seconds. **(1)**

....

(Total for Question 2 is 3 marks)

3 A particle moves so that its displacement from a fixed point is $s = 20t - 4t^2$ metres after t seconds. (a) Find the velocity function and hence find the time(s) when the particle is instantaneously at rest. (b) Find the acceleration and state whether the particle is speeding up or slowing down at t = 3 seconds. Give a short reason.

(a) Find v and the time(s) when v = 0. **(3)**

....

(b) Find a and state whether the particle is speeding up or slowing down at t = 3 s, with a brief reason. **(4)**

....

(Total for Question 3 is 7 marks)

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4 A model rocket moves vertically so that its displacement above the launch pad in metres is $s = t^3 - 4t^2 + t$ for time t seconds after launch. (a) Find the velocity function v = ds/dt. (b) Find the acceleration at t = 2 seconds and state, with a reason, whether the rocket is speeding up or slowing down at that instant.

(a) Find the velocity function v = ds/dt for the rocket. **(2)**

....

(b) Find the acceleration at t = 2 s and state whether the rocket is speeding up or slowing down at t = 2 s, with a brief reason. **(4)**

....

(Total for Question 4 is 6 marks)

5 A commuter walking along a straight platform has displacement $s = 50 - 5t - t^2$ metres from a fixed bench after t seconds. Find the time when the commuter is instantaneously at rest and state the direction of motion immediately after that time. Give a short reason.

(Total for Question 5 is 4 marks)

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6 A particle has displacement $s = 0.5t^3 - 3t^2 + 2t$ metres after t seconds. (a) Find expressions for velocity v and acceleration a. (b) Find the times $t \geq 0$ when the particle is instantaneously at rest. (c) For each time found, state the sign of the acceleration and the immediate change in speed, with a brief reason.

(a) Find v and a. **(2)**

....

(b) Find times $t \geq 0$ when $v = 0$. **(2)**

....

(c) For each time found, state the sign of a and whether the particle is momentarily speeding up or slowing down, with a short reason. **(2)**

(Total for Question 6 is 6 marks)

7 A motorbike's displacement along a straight road is $s = -2t^3 + 15t^2 - 24t$ metres after t seconds. (a) Find the acceleration a(t). (b) Find the time(s) $t \geq 0$ when the acceleration is zero. (c) For one interval between these times, state whether acceleration is positive or negative and give a brief implication for the motorbike's motion.

(a) Find the acceleration function a(t). **(2)**

....

(b) Find times $t \geq 0$ when acceleration is zero. **(2)**

....

(c) State the sign of acceleration for $t < 2.5$ and give a brief implication for the motorbike's motion in that interval. **(3)**

(Total for Question 7 is 7 marks)

8 An experimental glider has displacement $s = 4t^2 - t^3 + 6t$ metres from a reference point after t seconds. (a) Find v and a. (b) Find the time(s) $t \geq 0$ when the glider is instantaneously at rest. (c) For each such time give the acceleration value and a one line interpretation of the motion immediately after that instant.

(a) Find expressions for v and a. **(2)**

....

(b) Find times $t \geq 0$ when $v = 0$. **(3)**

....

(c) For each time found, give a value of a and a one line interpretation of the motion immediately after that instant. **(2)**

(Total for Question 8 is 7 marks)

Mark scheme · IG.M19 Calculus in Kinematics: Displacement, Velocity and Acceleration

Question 1

- **M1** differentiates s with respect to t, eg writes $d/dt(4t^2)$ or 8t seen
- **A1** v = 8t cao
- **Answer: v = 8t**

Question 2

- (a) **M1** differentiates each term, e.g. $9t^2$ and -5 seen
- (a) **A1** $v = 9t^2 - 5$ cao
- **(a) Answer: $v = 9t^2 - 5$**
- (b) **B1** $v(2) = 9(2)^2 - 5 = 31$ cao
- **(b) Answer: 31 m/s**

Question 3

- (a) **M1** differentiates s to find v = 20 - 8t or equivalent
- (a) **M1** sets 20 - 8t = 0 and rearranges to 8t = 20
- (a) **A1** t = 2.5 seconds cao
- **(a) Answer: v = 20 - 8t; particle at rest at t = 2.5 s**
- (b) **M1** differentiates v to find a = dv/dt = -8
- (b) **M1** substitutes t = 3 into v and/or uses sign of v at t = 3 or evaluates a at t = 3 (a is constant) to compare signs
- (b) **A1** a = -8 m/s^2 cao
- (b) **A1** particle is slowing down at t = 3 s, with correct reason linking signs, eg velocity at t = 3 is 20 - 24 = -4 m/s and acceleration is negative so speed is increasing in the negative direction meaning the magnitude of velocity is decreasing, or simply that velocity and acceleration have same/opposite sign explained correctly
- **(b) Answer: a = -8 m/s^2; at t = 3 s the particle is slowing down**

Question 4

- (a) **M1** differentiates terms of s to form v, eg $3t^2 - 8t + 1$ seen
- (a) **A1** $v = 3t^2 - 8t + 1$ cao
- **(a) Answer: $v = 3t^2 - 8t + 1$**
- (b) **M1** differentiates v to get a = 6t - 8
- (b) **M1** substitutes t = 2 to find a(2) = 4
- (b) **A1** a = 4 m/s^2 cao
- (b) **A1** correct statement about speeding up or slowing down with reason, eg v(2) = 3(4) - 16 + 1 = -3 m/s so velocity is negative while acceleration is positive, therefore the rocket is slowing down at t = 2 s, cao
- **(b) Answer: a = 4 m/s^2; the rocket is slowing down at t = 2 s, because v(2) = -3 m/s and a is positive so velocity and acceleration have opposite signs**

Question 5

- **M1** differentiates s to get v = -5 - 2t or equivalent
- **M1** sets -5 - 2t = 0 and solves for t, e.g. 2t = -5
- **A1** t = -2.5 s cao, or rejects as not physically valid if only $t \geq 0$ allowed and gives correct alternative statement
- **A1** direction immediately after that time: for physical $t \geq 0$ there is no positive time when at rest, or if candidate accepts negative time then for t just greater than -2.5 v is negative so motion is in the negative direction, with a concise correct reason
- **Answer: v = -5 - 2t so v = 0 at t = -2.5 s (not in the physical domain $t \geq 0$); therefore the commuter is never instantaneously at rest for $t \geq 0$. If negative time is considered, immediately after t = -2.5 s the velocity is negative, so motion is in the negative direction.**

Question 6

- (a) **M1** differentiates s to get $v = 1.5t^2 - 6t + 2$ or equivalent
- (a) **A1** $a = 3t - 6$ cao
- **(a) Answer: $v = 1.5t^2 - 6t + 2$; $a = 3t - 6$**
- (b) **M1** sets $1.5t^2 - 6t + 2 = 0$ and attempts to solve, e.g. multiply by 2 to $3t^2 - 12t + 4 = 0$
- (b) **A1** $t = (12 \pm \sqrt{144 - 48})/6 = (12 \pm \sqrt{96})/6 = (12 \pm 4*\sqrt{6})/6$, giving $t = 2 \pm (2/3)\sqrt{6}$, both values evaluated and non-negative, cao
- **(b) Answer: $t = 2 \pm (2/3)\sqrt{6}$ seconds (both are ≥ 0)**
- (c) **M1** evaluates a at each time correctly, recognising sign, eg $a(0.367) = 3(0.367) - 6 < 0$ and $a(3.633) = 3(3.633) - 6 > 0$
- (c) **A1** states correctly: at the smaller root $a < 0$ so since $v = 0$ and $a < 0$ the particle will start moving in the negative direction, speed will increase in the negative direction (interpreted as speeding up if velocity and acceleration same sign, here acceleration negative and velocity changes to negative so speed increases), and at the larger root $a > 0$ so particle will start moving in the positive direction and speed will increase in the positive direction, cao
- **(c) Answer: At t approx 0.367 s, $a < 0$ so the particle will start accelerating in the negative direction and speed will increase in that direction. At t approx 3.633 s, $a > 0$ so it will accelerate in the positive direction and speed will increase in the positive direction.**

Question 7

- (a) **M1** differentiates twice, eg $v = -6t^2 + 30t - 24$ and $a = -12t + 30$ shown
- (a) **A1** $a = -12t + 30$ cao
- **(a) Answer: $a = -12t + 30$**
- (b) **M1** sets $-12t + 30 = 0$ and solves to give $t = 30/12$
- (b) **A1** $t = 2.5$ seconds cao
- **(b) Answer: $t = 2.5$ s**
- (c) **M1** evaluates a for $t < 2.5$, e.g. picks $t = 0$ or states $-12t + 30 > 0$ when $t < 2.5$
- (c) **M1** states sign correctly, eg $a > 0$ for $t < 2.5$
- (c) **A1** gives correct implication, eg motorbike is accelerating (speed increasing) while $a > 0$, with a brief correct reason linking sign of a to change in speed
- **(c) Answer: For $t < 2.5$ s, $a > 0$, so the motorbike is accelerating (its speed is increasing) in that interval.**

Question 8

- (a) **M1** differentiates s to get $v = 8t - 3t^2 + 6$ or equivalent and $a = 8 - 6t$
- (a) **A1** $v = 8t - 3t^2 + 6$ and $a = 8 - 6t$ cao
- **(a) Answer: $v = 8t - 3t^2 + 6$; $a = 8 - 6t$**
- (b) **M1** sets $8t - 3t^2 + 6 = 0$ and attempts rearrangement, e.g. $3t^2 - 8t - 6 = 0$
- (b) **M1** uses quadratic formula or factorises correctly to find roots
- (b) **A1** $t = (8 \pm \sqrt{64 + 72})/(6) = (8 \pm \sqrt{136})/6 = (8 \pm 2*\sqrt{34})/6$, giving two real roots, numeric approximations t approx -0.215 (reject) and t approx 3.615 s, so $t = 3.615$ s (to 3 dp) accepted cao
- **(b) Answer: t approx 3.615 s (the only non-negative root)**
- (c) **M1** evaluates a at the non-negative root correctly, e.g. $a(3.615) = 8 - 6(3.615)$ approx -13.69
- (c) **A1** gives correct interpretation: a is negative so after coming to rest the glider will accelerate in the negative direction and speed will increase in that direction, cao
- **(c) Answer: a(3.615) approx -13.69 m/s^2; since $a < 0$ the glider will begin to move in the negative direction and its speed in that direction will increase immediately after $t = 3.615$ s.**