



IG.M20 Histograms and Frequency Density

EDEXCEL 4MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / ____

Answer ALL questions. Show all your working.

- 1** A grouped frequency table records heights of 50 students in cm: $140 \leq h < 150$: 5, $150 \leq h < 160$: 12, $160 \leq h < 170$: 18, $170 \leq h < 180$: 10, $180 \leq h < 190$: 5. Calculate the frequency density for the class $150 \leq h < 160$.

(Total for Question 1 is 1 mark)

- 2** From the same grouped table as Question 1, calculate the frequency density for the class $160 \leq h < 170$.

(Total for Question 2 is 1 mark)

- 3** A class $120 \leq x < 140$ has frequency 6 and a class width 20. Calculate its frequency density and give units used to describe the vertical axis on a histogram of x .

(Total for Question 3 is 2 marks)

- 4** A histogram will be drawn for the distribution: class $0 \leq t < 5$ freq 8, $5 \leq t < 15$ freq 20, $15 \leq t < 20$ freq 12. Calculate the frequency density for $5 \leq t < 15$ (note class width is not 10 for every class).

(Total for Question 4 is 2 marks)

- 5 A grouped table for speeds (km/h) shows: $0 \leq s < 20$: frequency 3 (width 20), $20 \leq s < 40$: frequency 12 (width 20), $40 \leq s < 60$: frequency 25 (width 20). What height should the bar for $40 \leq s < 60$ have on a histogram where vertical axis is frequency density?

(Total for Question 5 is 2 marks)

- 6 A student draws a histogram for daily rainfall (mm) with unequal widths. The class $0 \leq r < 2$ has area 4 on the histogram. The class $2 \leq r < 5$ has frequency 9 and width 3. Calculate the frequency density for $2 \leq r < 5$ and then the height of the bar. Show your working.

(Total for Question 6 is 3 marks)

- 7 A histogram shows the following bars (frequency densities shown): class $10 \leq x < 20$ height 0.5, class $20 \leq x < 30$ height 1.0, class $30 \leq x < 50$ height 0.6. Calculate the frequency in the class $30 \leq x < 50$ by using area of the bar.

(Total for Question 7 is 3 marks)

- 8 Complete the following: a histogram for exam marks has bars for classes $0 \leq m < 10$, $10 \leq m < 30$, $30 \leq m < 50$. The class $10 \leq m < 30$ has frequency 40 and width 20. The histogram shows the height of this bar as 2.0. Using this, calculate the total number of students represented by the three classes if the other bars have heights 1.5 (width 10) and 1.8 (width 20) respectively. Show working for each class area.

(Total for Question 8 is 4 marks)

- 9** A histogram bar for the interval $50 \leq y < 70$ has height 0.9. Estimate the frequency in the sub-interval $60 \leq y < 70$ (the upper half of that class) assuming uniform distribution within the class. State your method.

(Total for Question 9 is 3 marks)

- 10** The histogram below (data described) has a bar $0 \leq z < 4$ with height 1.5, a bar $4 \leq z < 7$ with height 2.4, and a bar $7 \leq z < 10$ with height 1.2. Use these to complete the frequency column of the table: classes 0-4, 4-7, 7-10. Give each frequency as an integer.

(Total for Question 10 is 3 marks)

- 11** A histogram is given for weights of parcels with unequal class widths. The class $0 \leq w < 2$ has frequency 8, $2 \leq w < 5$ has unknown frequency f , and $5 \leq w < 10$ has frequency 30. The histogram shows heights: $0 \leq w < 2$ height 4, $2 \leq w < 5$ height 3, $5 \leq w < 10$ height 6. Complete the frequency f using areas and show steps.

(Total for Question 11 is 4 marks)

- 12** A histogram representing age groups has classes $0 \leq a < 10$, $10 \leq a < 30$, $30 \leq a < 60$. The heights are 0.8, 0.6 and 0.4 respectively. Estimate the total population represented by the histogram and explain briefly why heights decrease as class width increases in some histograms.

(Total for Question 12 is 4 marks)

- 13** A histogram shows a bar for $100 \leq x < 140$ with height 0.75. A researcher asks for the estimated number in $110 \leq x < 120$. Explain how to estimate this number and calculate it, assuming uniform distribution within the class.

(Total for Question 13 is 4 marks)

- 14** A frequency table is partially blank. Classes: $0 \leq v < 5$ width 5 density 0.6, $5 \leq v < 12$ width 7 density unknown with frequency 21, $12 \leq v < 20$ width 8 density 1.0. Complete the missing density for $5 \leq v < 12$ and the total frequency for all classes.

(Total for Question 14 is 3 marks)

- 15** Explain briefly why, when drawing a histogram from a grouped frequency table with unequal class widths, the vertical axis must show frequency density rather than frequency. Give a short numerical example to support your explanation.

(Total for Question 15 is 3 marks)

Mark scheme · IG.M20 Histograms and Frequency Density

Question 1

- **B1** frequency density = frequency / class width = $12 / 10 = 1.2$ cao
- **Answer: 1.2**

Question 2

- **B1** frequency density = $18 / 10 = 1.8$ cao
- **Answer: 1.8**

Question 3

- **M1** finds frequency density = $6 / 20$ or writes correct division
- **A1** frequency density = 0.3 (per unit on x) or 0.3 per cm, cao
- **Answer: 0.3, units: frequency density in 'frequency per unit x' (e.g. per cm)**

Question 4

- **M1** identifies correct class width = 10 and divides 20 by 10
- **A1** frequency density = 2.0 cao
- **Answer: 2.0**

Question 5

- **M1** divides 25 by class width 20
- **A1** height = 1.25 (frequency per km/h) cao
- **Answer: 1.25**

Question 6

- **M1** uses density = frequency / width: density = $9 / 3$
- **A1** frequency density = 3.0 cao
- **A1** height = 3.0 (since height equals frequency density) cao
- **Answer: frequency density = 3.0, height = 3.0**

Question 7

- **M1** uses area = height * width with width = $50 - 30 = 20$
- **M1** calculates area = $0.6 * 20 = 12$
- **A1** frequency = 12 (people/items) cao
- **Answer: 12**

Question 8

- **M1** calculates frequency for first class from area: $1.5 * 10 = 15$
- **M1** uses given for middle class: $2.0 * 20 = 40$
- **M1** calculates frequency for third class: $1.8 * 20 = 36$
- **A1** total = $15 + 40 + 36 = 91$ cao
- **Answer: frequencies: 15, 40, 36. Total = 91**

Question 9

- **M1** uses area method and recognises width of full class = 20 and sub-interval width = 10
- **M1** calculates full class frequency = $0.9 * 20 = 18$, then takes half for 10 width = 9
- **A1** estimate = 9 cao
- **Answer: 9**

Question 10

- **M1** uses area = height * width for each class
- **M1** calculates areas: $1.5 \times 4 = 6$; $2.4 \times 3 = 7.2$; $1.2 \times 3 = 3.6$
- **A1** gives frequencies 6, 7.2 (accept 7.2) and 3.6 (accept 3.6) and if integers required rounding to nearest whole numbers 6, 7, 4 cao
- **Answer: frequencies (exact): 6, 7.2, 3.6. As integers (nearest): 6, 7, 4**

Question 11

- **M1** uses area = height * width to find frequencies for known bars: first and third
- **M1** calculates first area = $4 \times 2 = 8$ and third gives $6 \times 5 = 30$
- **M1** sets $f = \text{height} \times \text{width}$ for middle class: $f = 3 \times (5 - 2) = 3 \times 3$
- **A1** $f = 9$ cao
- **Answer: $f = 9$**

Question 12

- **M1** calculates areas: $0.8 \times 10 = 8$, $0.6 \times 20 = 12$, $0.4 \times 30 = 12$
- **M1** adds correctly to give total 32
- **M1** gives a correct reason linking height to frequency density and class width, e.g. density = frequency/width so wider classes often have lower heights for same frequency, or to keep area equal to frequency
- **A1** total = 32 and sensible explanation about density and width, cao
- **Answer: Total = 32, explanation: heights are frequency densities; density = frequency/width so if class width increases for similar frequencies the height falls to keep area equal to frequency**

Question 13

- **M1** states method: find full class frequency by area then take proportion corresponding to sub-interval width
- **M1** calculates full class width = 40 and full frequency = $0.75 \times 40 = 30$
- **M1** calculates sub-interval width = 10 and proportional frequency = $30 \times (10/40) = 7.5$
- **A1** gives estimate 7.5 and comment that this may be rounded or reported as 7 or 8, cao
- **Answer: Estimate = 7.5 (about 8 people if rounded)**

Question 14

- **M1** uses density = frequency / width to find density for middle class: $21 / 7$
- **M1** calculates densities and frequencies for other classes: first freq = $0.6 \times 5 = 3$; third freq = $1.0 \times 8 = 8$
- **A1** density for $5 \leq v < 12 = 3.0$ and total frequency = $3 + 21 + 8 = 32$ cao
- **Answer: middle density = 3.0, total frequency = 32**

Question 15

- **M1** states that area of bar must represent frequency, so height must be frequency/width when widths vary
- **M1** gives a correct short numerical example, e.g. class A width 2 freq 6 density 3, class B width 6 freq 18 density 3 so equal densities give proportional areas
- **A1** concludes that using frequency as height would misrepresent data when widths differ, cao
- **Answer: Because area represents frequency. Example: class width 2 freq 6 gives density 3, width 6 freq 18 gives density 3; plotting densities gives areas $2 \times 3 = 6$ and $6 \times 3 = 18$. If heights were frequencies, bars would be 6 and 18 and areas would be 12 and 108, misrepresenting counts.**