HIGHER

2.2 Functions: Composite and Inverse Functions: 2-hour Pack

Total Marks

EDEXCEL 4MA1 · Calculator allowed · about 120 minutes

Name: _____ Date: ____ / ____ / _____

Answer ALL questions. Show all your working.

1 $f(x) = 4x + 7$
$g(x) = x^2 + 2x + 1$
Solve $fg(x) = 107$.

(Total for Question 1 is 4 marks)

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2 $p(x) = x^2 - 1$
$q(x) = -2x + 1$
Find $qp(x)$. Give your answer in its simplest form.

(Total for Question 2 is 3 marks)

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3 $g(x) = 3x^2 - 3$
$h(x) = x + 5$
Find $gg(-2)$.

(Total for Question 3 is 3 marks)

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4 $f(x) = 2x - 2$

(a) Find $f^{-1}(x)$.

(b) Solve $f(x) = f^{-1}(x)$.

(Total for Question 4 is 4 marks)

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5 $f(x) = 5x + 2$

Find $f^{-1}(12)$.

(Total for Question 5 is 2 marks)

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6 $f(x) = 3x + 7$

$g(x) = x^2 - 3$

Solve $fg(x) = 73$.

(Total for Question 6 is 4 marks)

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7 $f(x) = -3x + 3$

$g(x) = -2x - 1$

Find $fg(x)$. Give your answer in its simplest form.

(Total for Question 7 is 3 marks)

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8 $g(x) = 4x - 2$
$h(x) = 2x^2 + 1$
Find $gh(4)$.

(Total for Question 8 is 3 marks)

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9 $f(x) = -2x - 3$
(a) Find $f^{-1}(x)$.
(b) Solve $f(x) = f^{-1}(x)$.

(Total for Question 9 is 4 marks)

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10 $f(x) = 4x + 4$
Find $f^{-1}(x)$.

(Total for Question 10 is 3 marks)

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11 $f(x) = -2x + 6$
Find $f^{-1}(2)$.

(Total for Question 11 is 2 marks)

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12 $f(x) = x - 1$
$g(x) = x^2 - 2$
Find $fg(x)$. Give your answer in its simplest form.

(Total for Question 12 is 3 marks)

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13 $p(x) = x - 4$
$q(x) = x^2 - 3$
Find $pq(2)$.

(Total for Question 13 is 3 marks)

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14 $f(x) = 4x + 3$
(a) Find $f^{-1}(x)$.
(b) Solve $f(x) = f^{-1}(x)$.

(Total for Question 14 is 4 marks)

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15 $h(x) = (4x + 6)/(x + 2)$, for $x \neq -2$
Find $h^{-1}(x)$.

(Total for Question 15 is 5 marks)

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16 $f(x) = 5x - 1$

Find $f^{-1}(6)$.

(Total for Question 16 is 2 marks)

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17 $f(x) = 4x + 1$

$g(x) = x^2 - 2$

Solve $fg(x) = 93$.

(Total for Question 17 is 4 marks)

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18 $f(x) = -2x - 1$

$g(x) = 2x + 4$

Find $gf(x)$. Give your answer in its simplest form.

(Total for Question 18 is 3 marks)

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19 $f(x) = -3x + 1$

$g(x) = x^2 - 5$

Find $gf(-3)$.

(Total for Question 19 is 3 marks)

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20 $f(x) = 2x + 2$

(a) Find $f^{-1}(x)$.

(b) Solve $f(x) = f^{-1}(x)$.

(Total for Question 20 is 4 marks)

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21 $g(x) = 3x - 2$

$h(x) = -3x - 9$

Find $hg(x)$. Give your answer in its simplest form.

(Total for Question 21 is 3 marks)

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22 $f(x) = 2x^2 - 1$

$g(x) = -x + 2$

Find $ff(0)$.

(Total for Question 22 is 3 marks)

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23 Define $f(x) = 1/(x + 1)$ for $x \neq -1$. For the function f in this context, find $f(f(1))$ and show your working.

(Total for Question 23 is 3 marks)

24 Let $f(x) = x^2 + 1$, defined for $x \geq 0$ only. Find $f^{-1}(x)$ for this restricted function by swapping variables and rearranging, and state the domain for f^{-1} in this context.

(Total for Question 24 is 3 marks)

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Mark scheme · 2.2 Functions: Composite and Inverse Functions: 2-hour Pack

Question 1

$fg(x) = 4x^2 + 8x + 11$	**M1**
rearranges $4x^2 + 8x + 11 = 107$ to $4x^2 + 8x - 96 = 0$ or isolates the squared term	**M1**
takes both square roots or factorises	**M1**
$x = -6$ or $x = 4$	**A1**

Answer: $x = -6$ or $x = 4$

Question 2

substitutes the inner function into the outer: $q(p(x)) = q(x^2 - 1)$	**M1**
expands brackets and collects like terms	**M1**
$qp(x) = -2x^2 + 3$ cao	**A1**

Answer: $qp(x) = -2x^2 + 3$

Question 3

$g(-2) = 9$	**M1**
$g(9)$	**M1**
$gg(-2) = 240$	**A1**

Answer: $gg(-2) = 240$

Question 4

$y = 2x - 2$ and rearranges for x	**M1**
$f^{-1}(x) = (x + 2)/2$	**A1**
sets $2x - 2 = (x + 2)/2$	**M1**
$x = 2$	**A1**

Answer: (a) $f^{-1}(x) = (x + 2)/2$; (b) $x = 2$

Question 5

solves $5x + 2 = 12$, or finds $f^{-1}(x) = (x - (2))/5$	**M1**
$f^{-1}(12) = 2$	**A1**

Answer: $f^{-1}(12) = 2$

Question 6

$fg(x) = 3x^2 - 2$	**M1**
rearranges $3x^2 - 2 = 73$ to $3x^2 - 75 = 0$ or isolates the squared term	**M1**
takes both square roots or factorises	**M1**
$x = -5$ or $x = 5$	**A1**

Answer: $x = -5$ or $x = 5$

Question 7

substitutes the inner function into the outer: $f(g(x)) = f(-2x - 1)$	**M1**
expands brackets and collects like terms	**M1**
$fg(x) = 6x + 6$ cao	**A1**

Answer: $fg(x) = 6x + 6$

Question 8

$h(4) = 33$	**M1**
$g(33)$	**M1**
$gh(4) = 130$	**A1**

Answer: $gh(4) = 130$

Question 9

$y = -2x - 3$ and rearranges for x	**M1**
$f^{-1}(x) = (-x - 3)/2$	**A1**
sets $-2x - 3 = (-x - 3)/2$	**M1**
$x = -1$	**A1**

Answer: (a) $f^{-1}(x) = (-x - 3)/2$; (b) $x = -1$

Question 10

writes $y = 4x + 4$ and swaps x and y (or rearranges for x)	**M1**
isolates the new subject: $y = (x - 4)/4$	**M1**
$f^{-1}(x) = (x - 4)/4$	**A1**

Answer: $f^{-1}(x) = (x - 4)/4$

Question 11

solves $-2x + 6 = 2$, or finds $f^{-1}(x) = (x - (6))/-2$	**M1**
$f^{-1}(2) = 2$	**A1**

Answer: $f^{-1}(2) = 2$

Question 12

substitutes the inner function into the outer: $f(g(x)) = f(x^2 - 2)$	**M1**
expands brackets and collects like terms	**M1**
$fg(x) = x^2 - 3$ cao	**A1**

Answer: $fg(x) = x^2 - 3$

Question 13

$q(2) = 1$	**M1**
$p(1)$	**M1**
$pq(2) = -3$	**A1**

Answer: $pq(2) = -3$

Question 14

$y = 4x + 3$ and rearranges for x	**M1**
$f^{-1}(x) = (x - 3)/4$	**A1**
sets $4x + 3 = (x - 3)/4$	**M1**
$x = -1$	**A1**

Answer: (a) $f^{-1}(x) = (x - 3)/4$; (b) $x = -1$

Question 15

$y = (4x + 6)/(x + 2)$, so $y(x + 2) = 4x + 6$	**M1**
expands and collects the x terms on one side	**M1**
factorises out x and divides	**M1**
swaps x and y	**M1**
$h^{-1}(x) = (-2x + 6)/(x - 4)$	**A1**

Answer: $h^{-1}(x) = (-2x + 6)/(x - 4)$

Question 16

solves $5x - 1 = 6$, or finds $f^{-1}(x) = (x - (-1))/5$	**M1**
$f^{-1}(6) = 7/5$	**A1**

Answer: $f^{-1}(6) = 7/5$

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Question 17

$fg(x) = 4x^2 - 7$	**M1**
rearranges $4x^2 - 7 = 93$ to $4x^2 - 100 = 0$ or isolates the squared term	**M1**
takes both square roots or factorises	**M1**
$x = -5$ or $x = 5$	**A1**

Answer: x = -5 or x = 5

Question 18

substitutes the inner function into the outer: $g(f(x)) = g(-2x - 1)$	**M1**
expands brackets and collects like terms	**M1**
$gf(x) = -4x + 2$ cao	**A1**

Answer: gf(x) = -4x + 2

Question 19

$f(-3) = 10$	**M1**
$g(10)$	**M1**
$gf(-3) = 95$	**A1**

Answer: gf(-3) = 95

Question 20

$y = 2x + 2$ and rearranges for x	**M1**
$f^{-1}(x) = (x - 2)/2$	**A1**
sets $2x + 2 = (x - 2)/2$	**M1**
$x = -2$	**A1**

Answer: (a) $f^{-1}(x) = (x - 2)/2$; (b) x = -2

Question 21

substitutes the inner function into the outer: $h(g(x)) = h(3x - 2)$	**M1**
expands brackets and collects like terms	**M1**
$hg(x) = -9x - 3$ cao	**A1**

Answer: hg(x) = -9x - 3

Question 22

$f(0) = -1$	**M1**
$f(-1)$	**M1**
$ff(0) = 1$	**A1**

Answer: ff(0) = 1

Question 23

finds $f(1) = 1/2$ or equivalent	**M1**
substitutes $f(1)$ into f correctly, showing $f(1/2) = 1/(1/2 + 1)$	**M1**
$f(f(1)) = 2/3$ cao	**A1**

Answer: 2/3

Question 24

starts with $y = x^2 + 1$ and swaps x and y to $x = y^2 + 1$ or writes $x = y^2 + 1$	**M1**
rearranges to $y = \sqrt{x - 1}$ and states the principal (non-negative) root	**M1**
$f^{-1}(x) = \sqrt{x - 1}$ with domain $x \geq 1$ cao	**A1**

Answer: $f^{-1}(x) = \sqrt{x - 1}$, domain $x \geq 1$

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