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HIGHER

IG.M4 Functions: Composite and Inverse Functions

EDEXCEL 4MA1 · Calculator allowed · about 50 minutes

Total Marks

Name: ______________________________ Date: ____ / ____ / ______

Answer ALL questions. Show all your working.

1 Let f be the function defined by $f(x) = 2x + 3$. For the function f in this context, find $f(0)$.

(Total for Question 1 is 1 mark)

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2 The function g is defined by $g(x) = x^2 - 4$. For the function g in this question, find $g(3)$.

(Total for Question 2 is 1 mark)

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3 Let $f(x) = 3x - 1$ and $h(x) = 5$. For these functions, work out $f(h(2))$.

(Total for Question 3 is 2 marks)

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4 For functions $f(x) = x + 2$ and $g(x) = 2x$, find and simplify $g(f(x))$. State the composite as an algebraic expression in x for the functions named.

(Total for Question 4 is 2 marks)

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5 With the same $f(x) = x + 2$ and $g(x) = 2x$ as in Question 4, find and simplify $f(g(x))$.

(Total for Question 5 is 2 marks)

6 Let $f(x) = 4x - 5$ and $g(x) = x^2$. Work out fg(2) and gf(2), where fg(x) means f(g(x)) and gf(x) means g(f(x)).

(Total for Question 6 is 3 marks)

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7 Define $f(x) = 1/(x + 1)$ for $x \neq -1$. For the function f in this context, find f(f(1)) and show your working.

(Total for Question 7 is 3 marks)

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8 Let $f(x) = 2x + 1$ and $g(x) = x - 3$. Show that the composite $fg(x) = 2x - 5$, where fg(x) denotes f(g(x)).

(Total for Question 8 is 3 marks)

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9 The function f is defined by $f(x) = 3x + 2$. Find the inverse function $f^{-1}(x)$ by swapping variables and rearranging, and state $f^{-1}(x)$ as an algebraic expression.

(Total for Question 9 is 4 marks)

10 Find the inverse of the function $g(x) = (x - 4)/2$ by swapping variables and rearranging, for the function g named here. Give $g^{-1}(x)$ in simplest form.

(Total for Question 10 is 3 marks)

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11 Let $f(x) = 2x + 1$ and $g(x) = (x - 1)/2$. By inspection or composition, show whether g is the inverse of f by calculating $f(g(x))$ and $g(f(x))$. Name the conclusion for the functions in this question.

(Total for Question 11 is 3 marks)

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12 Let $f(x) = x^2 + 1$, defined for $x \geq 0$ only. Find $f^{-1}(x)$ for this restricted function by swapping variables and rearranging, and state the domain for f^{-1} in this context.

(Total for Question 12 is 3 marks)

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13 Given $f(x) = 2x + 3$ and $g(x) = x^2$, solve the equation $f(g(x)) = 11$ for x, where $f(g(x))$ denotes f composed with g, and show working.

(Total for Question 13 is 3 marks)

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14 Let $f(x) = (x + 2)/3$. Solve $f(x) = 5$ for x by rearrangement, showing a correct method and final value for the function named here.

(Total for Question 14 is 2 marks)

15 The function f is defined by $f(x) = x/2 - 7$. Find $f^{-1}(x)$ by swapping variables and rearranging, and give $f^{-1}(3)$.

(Total for Question 15 is 3 marks)

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16 Let $f(x) = x - 4$ and $g(x) = x/5$. Find and simplify $fg(x)$ and $gf(x)$ for the functions named, and state whether $fg = gf$ as functions.

(Total for Question 16 is 2 marks)

Mark scheme · IG.M4 Functions: Composite and Inverse Functions

Question 1

- **B1** $f(0) = 3$ cao
- **Answer: 3**

Question 2

- **B1** $g(3) = 5$ cao
- **Answer: 5**

Question 3

- **M1** evaluates $h(2) = 5$ and substitutes into f, or shows $f(5)$ seen
- **A1** $f(h(2)) = 14$ cao
- **Answer: 14**

Question 4

- **M1** substitutes $f(x)$ into g as $g(f(x)) = 2(f(x))$ or $2(x + 2)$
- **A1** $g(f(x)) = 2x + 4$ cao
- **Answer: $g(f(x)) = 2x + 4$**

Question 5

- **M1** substitutes $g(x)$ into f as $f(g(x)) = g(x) + 2$ or $x*2 + 2$
- **A1** $f(g(x)) = 2x + 2$ cao
- **Answer: $f(g(x)) = 2x + 2$**

Question 6

- **M1** evaluates $g(2) = 4$ and substitutes into f, or shows $f(g(2))$ process
- **M1** evaluates $f(2) = 3$ and substitutes into g, or shows $g(f(2))$ process
- **A1** $fg(2) = 11$ and $gf(2) = 9$ cao
- **Answer: $fg(2) = 11$; $gf(2) = 9$**

Question 7

- **M1** finds $f(1) = 1/2$ or equivalent
- **M1** substitutes $f(1)$ into f correctly, showing $f(1/2) = 1/(1/2 + 1)$
- **A1** $f(f(1)) = 2/3$ cao
- **Answer: 2/3**

Question 8

- **M1** substitutes $g(x)$ into f: $f(g(x)) = 2(g(x)) + 1$
- **M1** substitutes $g(x) = x - 3$ giving $2(x - 3) + 1$
- **A1** simplifies to $fg(x) = 2x - 5$ cao
- **Answer: $fg(x) = 2x - 5$**

Question 9

- **M1** starts with $y = 3x + 2$ and swaps x and y or writes $x = 3y + 2$
- **M1** rearranges correctly: $x - 2 = 3y$ or $y = (x - 2)/3$ seen
- **A1** $f^{-1}(x) = (x - 2)/3$ cao
- **B1** states domain restriction or checks composition eg $f(f^{-1}(x)) = x$, oe
- **Answer: $f^{-1}(x) = (x - 2)/3$**

Question 10

- **M1** starts with $y = (x - 4)/2$ and swaps x and y to $x = (y - 4)/2$ or equivalent
- **M1** rearranges to $y = 2x + 4$
- **A1** $g^{-1}(x) = 2x + 4$ cao
- **Answer: $g^{-1}(x) = 2x + 4$**

Question 11

- **M1** computes $f(g(x)) = 2((x - 1)/2) + 1$ and simplifies to x
- **M1** computes $g(f(x)) = (2x + 1 - 1)/2$ and simplifies to x
- **A1** concludes g is the inverse of f since both compositions give x cao
- **Answer: Both $f(g(x)) = x$ and $g(f(x)) = x$, so g is the inverse of f**

Question 12

- **M1** starts with $y = x^2 + 1$ and swaps x and y to $x = y^2 + 1$ or writes $x = y^2 + 1$
- **M1** rearranges to $y = \sqrt{x - 1}$ and states the principal (non-negative) root
- **A1** $f^{-1}(x) = \sqrt{x - 1}$ with domain $x \ge 1$ cao
- **Answer: $f^{-1}(x) = \sqrt{x - 1}$, domain $x \ge 1$**

Question 13

- **M1** writes $f(g(x)) = 2x^2 + 3$ and sets equal to 11
- **M1** solves $2x^2 + 3 = 11$ to get $x^2 = 4$
- **A1** $x = 2$ or $x = -2$ cao
- **Answer: $x = 2$ or $x = -2$**

Question 14

- **M1** multiplies both sides by 3 or shows rearrangement $3f(x) = x + 2$
- **A1** $x = 13$ cao
- **Answer: $x = 13$**

Question 15

- **M1** writes $y = x/2 - 7$ and swaps to $x = y/2 - 7$ or equivalent
- **M1** rearranges to $y = 2(x + 7)$ or $y = 2x + 14$
- **A1** $f^{-1}(x) = 2x + 14$ and $f^{-1}(3) = 20$ cao
- **Answer: $f^{-1}(x) = 2x + 14$; $f^{-1}(3) = 20$**

Question 16

- **M1** finds $fg(x) = f(g(x)) = g(x) - 4 = x/5 - 4$ and $gf(x) = g(f(x)) = (x - 4)/5$
- **A1** states $fg(x) = x/5 - 4$, $gf(x) = (x - 4)/5$ and concludes they are not equal as functions cao
- **Answer: $fg(x) = x/5 - 4$; $gf(x) = (x - 4)/5$. They are not equal as functions.**