



IG.M5 Matrices and Matrix Transformations

EDEXCEL 4MA1 · Calculator allowed · about 60 minutes

Total Marks

Name: _____ Date: ____ / ____ / ____

Questions 1 to 12 do not require full sentence answers. Questions 13 to 16 require full sentences when describing transformations. Allow 60 minutes.

- 1** In the context of matrix arithmetic for a plotting task, calculate the sum of the 2×2 matrices $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 \\ -2 & 0 \end{bmatrix}$.

(Total for Question 1 is 1 mark)

- 2** For a transformation exercise, calculate $A - B$ where $A = \begin{bmatrix} 7 & 2 \\ 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -1 \\ 5 & 2 \end{bmatrix}$.

(Total for Question 2 is 1 mark)

- 3** A gardener uses a 2×1 column matrix to record a plant position. Calculate 4 times the column matrix $v = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ to scale coordinates for an enlargement.

(Total for Question 3 is 2 marks)

- 4** In matrix transformations, calculate the product of 2×2 matrices $C = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ to combine two linear maps.

(Total for Question 4 is 2 marks)

- 5 Apply a linear transformation to a point. Calculate the image of point P with position vector $p = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ under the 2×2 matrix $T = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$.

(Total for Question 5 is 2 marks)

- 6 For a 2×2 matrix used in area-scaling, calculate the determinant of $M = \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$. State the determinant value which indicates area scale factor.

(Total for Question 6 is 2 marks)

- 7 Find the inverse of the non-singular 2×2 matrix $N = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ to reverse a linear transformation used on coordinates.

(Total for Question 7 is 3 marks)

- 8 Determine whether the 2×2 matrix $S = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$ is singular, for a coordinate transform used by Priya. If singular, give a brief reason.

(Total for Question 8 is 3 marks)

- 9 Calculate the product of a 2×2 transformation matrix and a column vector. Let $U = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (a rotation by 90 degrees) and $q = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$. Find Uq , the image of q under this rotation.

(Total for Question 9 is 2 marks)

- 10** Calculate the 2×2 matrix that represents a combined transformation of first scaling by 3 in all directions, then reflecting in the x-axis. Use matrices $S = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ for enlargement and $R = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ for reflection. Find the matrix product RS which maps a point after both transformations.

(Total for Question 10 is 2 marks)

- 11** Find the 2×2 matrix which represents a rotation of 180 degrees about the origin, and then use it to find the image of point with position vector $r = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$.

(Total for Question 11 is 3 marks)

- 12** Combine two 2×2 linear transformations. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (reflection in the line $y = x$) and $B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ (horizontal stretch by factor 2). Calculate the single 2×2 matrix that is equivalent to doing B then A (i.e. compute AB).

(Total for Question 12 is 4 marks)

- 13** Describe fully the single transformation that maps the square with vertices at $(1,1)$, $(2,1)$, $(2,2)$, $(1,2)$ to the square with vertices at $(-2, -2)$, $(-4, -2)$, $(-4, -4)$, $(-2, -4)$. Give the transformation as a combination of standard terms (scale factor, centre, rotation angle, reflection as appropriate) and any translation vector.

(Total for Question 13 is 4 marks)

- 14** Describe fully the single transformation that maps point A with position vector $\mathbf{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to A' with position vector $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$, and maps B with position vector $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to B' with position vector $\begin{bmatrix} -2 \\ 0 \end{bmatrix}$. Use standard transformation language (rotation, reflection, enlargement, translation) and state centre or angle as needed.

(Total for Question 14 is 5 marks)

- 15** Two transformations are combined: first reflect in the y-axis, then translate by vector $\mathbf{t} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$. Describe fully the single transformation that maps an arbitrary point (x,y) to its final image. Give the type of transformation, and if it is not a pure single linear transformation explain it as 'reflection then translation' and give the net effect on coordinates.

(Total for Question 15 is 5 marks)

- 16** A map on the plane is represented by the matrix $P = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ followed by the translation vector $\mathbf{u} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$. Describe fully the single transformation performed, and give the image of point (3, -1).

(Total for Question 16 is 5 marks)

Mark scheme · IG.M5 Matrices and Matrix Transformations

Question 1

- **B1** $A + B = \begin{bmatrix} 3 & 4 \\ 1 & 4 \end{bmatrix}$ cao
- **Answer:** $\begin{bmatrix} 3 & 4 \\ 1 & 4 \end{bmatrix}$

Question 2

- **B1** $A - B = \begin{bmatrix} 3 & 3 \\ -5 & -5 \end{bmatrix}$ cao
- **Answer:** $\begin{bmatrix} 3 & 3 \\ -5 & -5 \end{bmatrix}$

Question 3

- **M1** multiplies both entries by 4
- **A1** $4v = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$ cao
- **Answer:** $\begin{bmatrix} 8 \\ -4 \end{bmatrix}$

Question 4

- **M1** multiplies rows of C by columns of D to form product entries
- **A1** $CD = \begin{bmatrix} 3 & 6 \\ -3 & 12 \end{bmatrix}$ cao
- **Answer:** $\begin{bmatrix} 3 & 6 \\ -3 & 12 \end{bmatrix}$

Question 5

- **M1** performs matrix multiplication T p
- **A1** $T p = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ cao
- **Answer:** $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$

Question 6

- **M1** uses ad - bc method correctly
- **A1** determinant = 13 cao
- **Answer:** 13

Question 7

- **M1** calculates determinant correctly as 5
- **M1** forms adjugate matrix by swapping diagonal entries and negating off-diagonal entries
- **A1** $N^{-1} = (1/5)\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$ cao
- **Answer:** $(1/5)\begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$

Question 8

- **M1** calculates determinant correctly using ad - bc
- **M1** states determinant = 0 or shows rows are proportional
- **A1** concludes S is singular with correct reason, e.g. $\det = 0$ so no inverse exists
- **Answer:** Singular, because determinant = $2 \times 2 - 4 \times 1 = 4 - 4 = 0$

Question 9

- **M1** performs correct multiplication of U and q
- **A1** $U q = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ cao
- **Answer:** $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$

Question 10

- **M1** multiplies R and S in correct order R S
- **A1** $RS = \begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix}$ cao
- **Answer:** $\begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix}$

Question 11

- **M1** states or writes rotation by 180 degrees matrix as $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
- **M1** multiplies matrix by the column vector r
- **A1** image = $\begin{bmatrix} 2 \\ -5 \end{bmatrix}$ cao
- **Answer:** $\begin{bmatrix} 2 \\ -5 \end{bmatrix}$

Question 12

- **M1** multiplies A and B in the correct order A B
- **M1** computes entries correctly for first row
- **M1** computes entries correctly for second row
- **A1** A B = $\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ cao
- **Answer:** $\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$

Question 13

- **M1** identifies change in size: scale factor 2 or 2x enlargement or reduction
- **M1** identifies orientation change: rotation by 180 degrees or reflection equivalent
- **M1** gives correct centre or translation vector, e.g. maps centre (1.5, 1.5) to centre (-3, -3) so translation vector is (-4.5, -4.5) or states combined as enlargement with negative scale about origin
- **A1** fully correct description, for example: enlargement by scale factor 2 with centre at the origin combined with a reflection through the origin, equivalent to a rotation by 180 degrees and then scale factor 2, which maps the small square to the large one; or concise equivalent description with correct translation vector

Question 14

- **M1** identifies mapping of basis vectors implies linear map corresponding to matrix $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$ or recognises rotation and scale
- **M1** shows that a maps to (0, 2) and b maps to (-2, 0) consistent with a 90 degree rotation combined with scale factor 2
- **M1** states the centre is the origin and gives angle and scale factor, e.g. rotation by 90 degrees anticlockwise and enlargement by factor 2 about the origin
- **M1** explains that this is equivalent to multiplying by matrix $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$ or alternatively enlargement by 2 then rotation by 90 degrees
- **A1** full correct description, e.g. 'enlargement by factor 2 with centre at the origin combined with a rotation of 90 degrees anticlockwise about the origin' or equivalent wording

Question 15

- **M1** describes reflection in y-axis as mapping (x, y) to (-x, y)
- **M1** applies translation to give (-x + 3, y + 1)
- **M1** states that the combined map is not a single linear transformation but an isometry composed of reflection then translation
- **M1** gives final coordinate rule correctly as (x, y) $\rightarrow$ (3 - x, y + 1)
- **A1** full clear description that the mapping is: reflect in the y-axis, then translate by vector (3, 1), equivalently (x, y) maps to (3 - x, y + 1)

Question 16

- **M1** identifies P as reflection in the y-axis
- **M1** applies reflection to point (3, -1) to get (-3, -1)
- **M1** applies translation vector u to give final image (-1, -1)
- **M1** states combined transformation fully as reflection in the y-axis then translation by (2, 0) and notes it is not a single 2x2 linear map
- **A1** final image given and description complete, e.g. image of (3, -1) is (-1, -1) and the transformation is reflect in the y-axis then translate by (2, 0)